# SAINIK SCHOOL IMPHAL

## SUMMER VACATION

## 2025-26

## HOMEWORK/ASSIGNMENT/PROJECTS

# CLASS XII

**CLASS 12 MATHS**

**NCERT CHAPTER-2: INVERSE TRIGONOMETRIC FUNCTIONS (ITF)**

**How to test a graph is a function or not (vertical line test):**

Draw a vertical line to the graph ,if it intersect the graph only at one point ,then the graph is a function, otherwise it is not a function.

fig-1

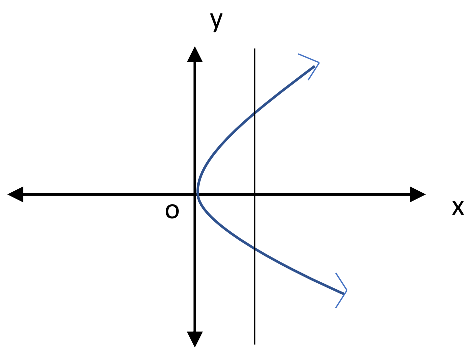
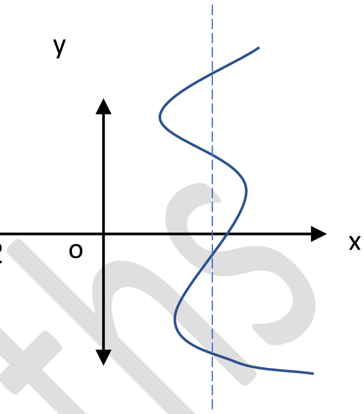



fig-2




The graph in fig-1 and fig-2 are not function.

**How to test a function is one-one or not (horizontal line test):**

Draw a horizontal line to the graph of function, if it intersect the graph only at one point, then the function is one-one , otherwise it is a many-one function.

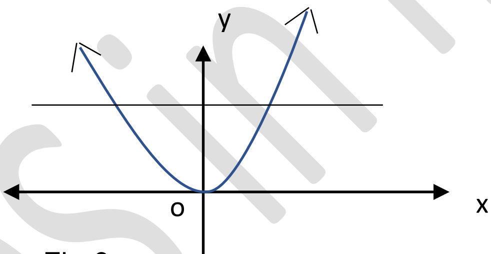



Fig-3

The function in Fig-3 is many-one.

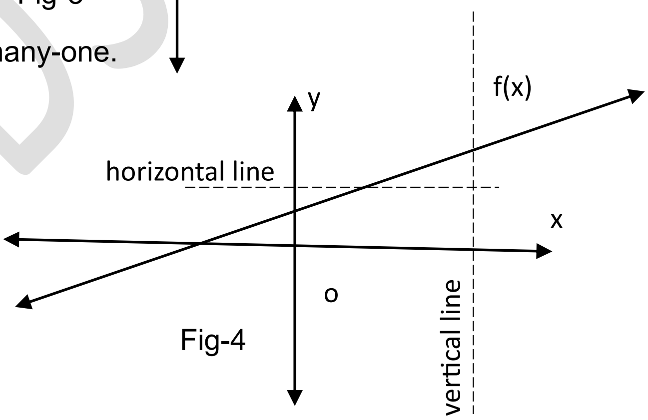



Fig-4

The function in Fig-4 is one-one onto.

The function in Fig-5 is one-one onto.

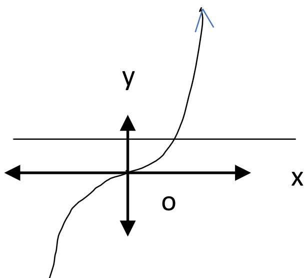



Fig-5

**Graph of inverse function**: Let a function $f(x) = x + 1$

Then $f^{-1}(x) = x - 1$

The graphs of $f(x)$ and $f^{-1}(x)$ are drawn as shown in fig-6.

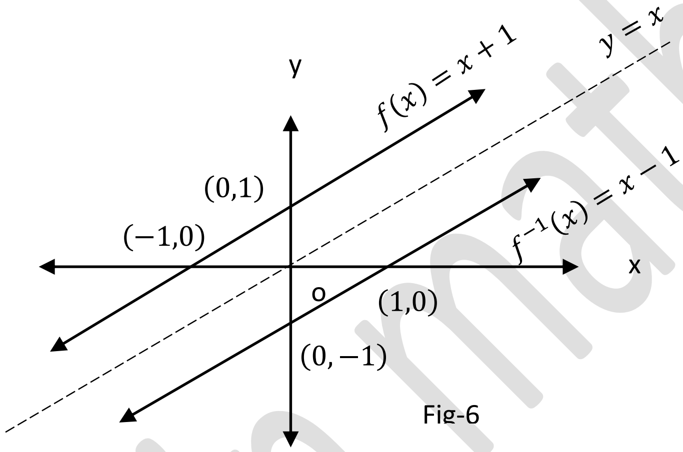
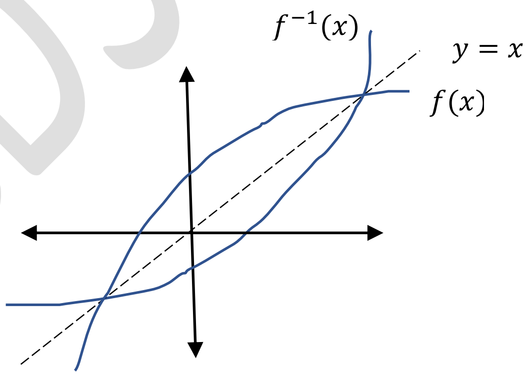



Fig-6

It is observed that the graph of $f^{-1}(x)$ is the reflection of graph of $f(x)$ with respect to line y $= x$ . Also the image of the point $(0,1)$ is the point $(1,0)$ and the image of the point $(-1,0)$ is the point $(0,-1)$ i.e. $x$ and $y$ co-ordinates of $f(x)$ acts as $y$ and $x$ co-ordinates of $f^{-1}(x)$.



**Domain and range of Trigonometric Functions:**

| Function | Domain | Range |
|---|---|---|
| $\sin x$ | R | [-1,1] |
| $\cos x$ | R | [-1,1] |
| $\tan x (= \frac{\sin x}{\cos x})$ | $R - (2n+1)\frac{\pi}{2}, n \in Z$ | $(-\infty, \infty)$ |
| $\cot x (= \frac{\cos x}{\sin x})$ | $R - n\pi, n \in Z$ | $(-\infty, \infty)$ |
| $\sec x (= \frac{1}{\cos x})$ | $R - (2n+1)\frac{\pi}{2}, n \in Z$ | $(-\infty, -1] \cup [1, \infty)$ |
| $\operatorname{cosec} x (= \frac{1}{\sin x})$ | $R - n\pi, n \in Z$ | $(-\infty, -1] \cup [1, \infty)$ |

**We know if $f: A \longrightarrow B$ ,then for $f^{-1}$ to exists $f$ must be one-one and onto.**

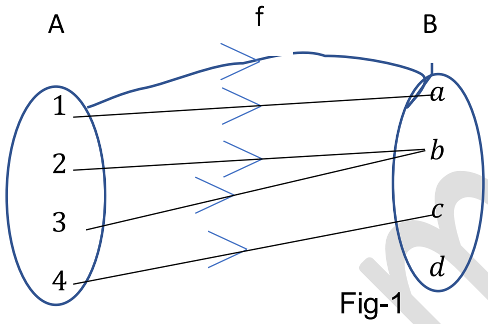
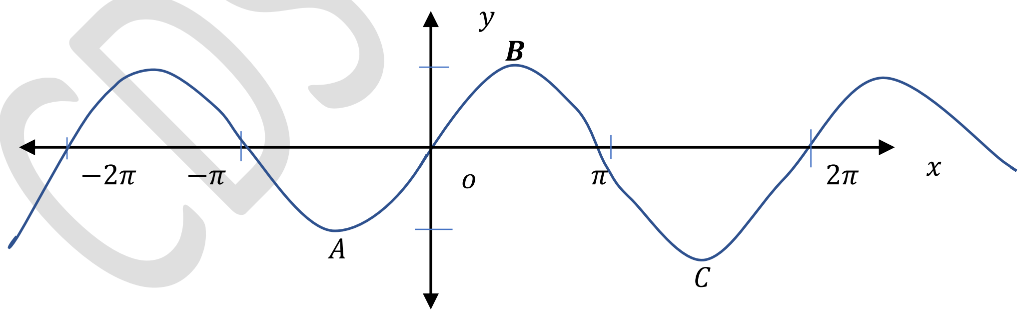


Fig-1

**Example1:** In Fig-1 $f$ is neither one-one nor onto and so $f^{-1}$ does not exists as $f^{-1}(b)$ has more than one image and $f^{-1}(d)$ is also not defined.

**Example2:** sin : $R \to R$ is neither one-one nor onto as $\sin 0 = \sin \pi = 0$

and range of $sine = [-1,1] \neq Co\ domain\ R$ and so inverse of sine function does not exist on R.




But it is clear from the graph of $y = \sin x$ that sine function is bijective on part AB of the graph for which domain is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and co-domain is $[-1, 1]$. Also, it is clear from the graph that sine is bijective for the branches $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$ etc. However, the branch $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is called the principal branch.

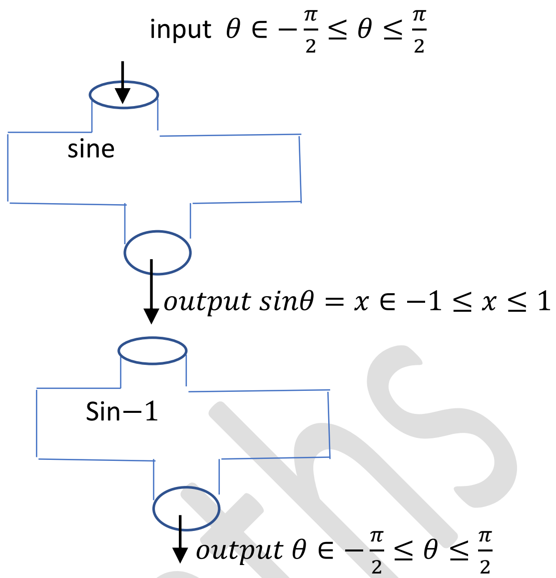
Thus $sine$: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$ is one-one and onto and so its inverse can be defined.

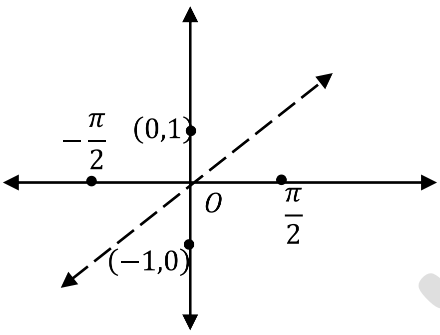
**Definition:** If $\sin\theta = x$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $-1 \le x \le 1$ , then $\theta$ is called sine inverse of $x$ and is written as $\theta = \sin^{-1} x$.

i.e $\sin^{-1}$: $[-1,1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

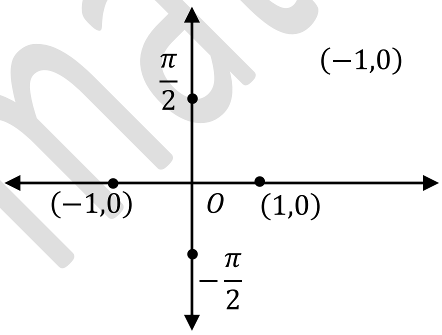
$\therefore \boldsymbol{\sin\theta = x \Leftrightarrow \theta = \sin^{-1} x}$

Similarly, by restricting domain and co-domain of other five trigonometric functions, they can be made One-one and onto and then their inverse can be defined.




Graph of bijective sine function




Graph of bijective sine function

**Domain and Range of Inverse Trigonometric Functions**

| Function | Domain | Range(principal branch) |
|---|---|---|
| $\sin^{-1} x$ | [-1,1] | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ |
| $\cos^{-1} x$ | [-1,1] | $[0,\pi]$ |
| $\tan^{-1} x$ | $(-\infty,\infty)$ | $(-\frac{\pi}{2},\frac{\pi}{2})$ |
| $\cot^{-1} x$ | $(-\infty,\infty)$ | $(0,\pi)$ |
| $\sec^{-1} x$ | $(-\infty,-1] \cup [1,\infty)$ | $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ |
| $\text{cosec}^{-1} x$ | $(-\infty,-1] \cup [1,\infty)$ | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ |

**Note: The value of an ITF which lies in the principal branch is called principal value of ITF.**

**Properties (Formulas) of ITF:**

1. **Cancellation laws:**

$\sin^{-1}(\sin x) = x$ , where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\cos^{-1}(\cos x) = x$ ,where $0 \le x \le \pi$

$\tan^{-1}(\tan x) = x$ ,where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\cot^{-1}(\cot x) = x$ , where $0 < x < \pi$

$\sec^{-1}(\sec x) = x$ , where $0 \le x \le \pi$ ,$x \ne \frac{\pi}{2}$

$\text{cosec}^{-1}(\text{cosec}\, x) = x$ , where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ ,$x \ne 0$

2. **Cancellation laws:**

$\sin(\sin^{-1} x) = x$ ,where $-1 \le x \le 1$

$\cos(\cos^{-1} x) = x$ ,where $-1 \le x \le 1$

$\tan(\tan^{-1} x) = x$ ,where $-\infty < x < \infty$

$\cot(\cot^{-1} x) = x$ ,where $-\infty < x < \infty$

$\sec(\sec^{-1} x) = x$ ,where $x \ge 1$ *or* $x \le -1$

$\text{cosec}(\text{cosec}^{-1} x) = x$ , where $x \ge 1$ *or* $x \le -1$

3.
$$\left.\begin{aligned} \sin^{-1}(-x) &= -\sin^{-1} x \text{ , where } -1 \le x \le 1 \\ \tan^{-1}(-x) &= -\tan^{-1} x \text{ , where } -\infty < x < \infty \\ \text{cosec}^{-1}(-x) &= -\text{cosec}^{-1} x \text{ , where } x \ge 1 \text{ or } x \le -1 \end{aligned}\right\} (odd\ functions)$$

$$\left.\begin{aligned} \cos^{-1}(-x) &= \pi - \cos^{-1}x \text{ ,where } -1 \le x \le 1 \\ \cot^{-1}(-x) &= \pi - \cot^{-1}x \text{ ,where } -\infty < x < \infty \\ \sec^{-1}(-x) &= \pi - \sec^{-1}x \text{ ,where } x \ge 1 \ or\ x \le -1 \end{aligned}\right\} (NENO\ functions)$$

4. $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}x$ , where $x \ge 1\ or\ x \le -1$

   $\cos^{-1}\frac{1}{x} = \sec^{-1}x$ , where $x \ge 1\ or\ x \le -1$

   $\tan^{-1}\frac{1}{x} = \cot^{-1}x$ , where $x > 0$

5. **Sum of complementary ITF** $(= \frac{\pi}{2})$**:**

   $\boldsymbol{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$ ,where $-1 \le x \le 1$

   $\boldsymbol{\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}}$ , where $-\infty < x < \infty$

   $\boldsymbol{\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}}$ , where $x \ge 1\ or\ x \le -1$

6. $$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\frac{x+y}{1-xy} & ,where\ x > 0, y > 0, xy < 1 \\ \tan^{-1}\frac{x+y}{1-xy} + \pi & ,where\ x > 0, y > 0, xy > 1 \\ \tan^{-1}\frac{x+y}{1-xy} - \pi & ,where\ x < 0, y < 0, xy > 1 \end{cases}$$

7. $$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\frac{x-y}{1+xy} & ,where\ x > 0, y > 0, xy > -1 \\ \tan^{-1}\frac{x-y}{1+xy} + \pi & ,where\ x > 0, y < 0, xy < -1 \\ \tan^{-1}\frac{x-y}{1+xy} - \pi & ,where\ x < 0, y > 0, xy < -1 \end{cases}$$

**8.** $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

9. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ ,where $-\frac{1}{2} \le x \le \frac{1}{2}$

10. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ ,where $\frac{1}{2} \le x \le 1$

11. $\tan^{-1}\frac{1+x}{1-x} = \frac{\pi}{4} + \tan^{-1}x$

12. $\tan^{-1}\frac{1-x}{1+x} = \frac{\pi}{4} - \tan^{-1}x$

13. $$2\tan^{-1}x = \begin{cases} \sin^{-1}\frac{2x}{1+x^2} \\ \cos^{-1}\frac{1-x^2}{1+x^2} \\ \tan^{-1}\frac{2x}{1-x^2} \end{cases}$$

14. $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$ ,where $x, y \in [-1,1]$

15. $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ , where $x, y \in [-1,1]$

16. $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$ , where $x, y \in [-1,1]$

17. $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$ , where $x, y \in [-1,1]$

**Deductions:**

$\mathbf{2}\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$ , where $-1 < x < 1$

$\mathbf{3}\tan^{-1}x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$ , where $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}(1-2x^2)$$

$$2\cos^{-1}x = \cos^{-1}(2x^2-1) = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

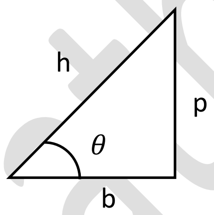
$$\{0 \le 2\cos^{-1}x \le \pi \Rightarrow 0 \le \cos^{-1}x \le \frac{\pi}{2} \Rightarrow 1 \ge x \ge 0 \Rightarrow 0 \le x \le 1\}$$

**Magic Triangle( Used for quick conversion of any inverse function to another):**

$\tan\theta = \frac{p}{b} \Rightarrow \theta = \tan^{-1}\left(\frac{p}{b}\right)$

$\sin\theta = \frac{p}{h} \Rightarrow \theta = \sin^{-1}\left(\frac{p}{h}\right)$

$\therefore \sin^{-1}\left(\frac{p}{h}\right) = \tan^{-1}\left(\frac{p}{b}\right)$ **etc.**




**Important Trigonometric Substitutions:**

| S No | Algebraic expression | Substitution for $x$ |
|---|---|---|
| 1 | $1-x^2$ | $x = \sin\theta$ |
| 2 | $a^2-x^2$ | $x = a\sin\theta$ |
| 3 | $1+x^2$ | $x = \tan\theta$ |
| 4 | $a^2+x^2$ | $x = a\tan\theta$ |
| 5 | $x^2-1$ | $x = \sec\theta$ |
| 6 | $x^2-a^2$ | $x = a\sec\theta$ |
| 7 | $\frac{\sqrt{a+x}\pm\sqrt{a-x}}{\sqrt{a+x}\mp\sqrt{a-x}}$ | $x = a\cos2\theta$ |
| 8 | $\frac{\sqrt{1+x^2}\pm\sqrt{1-x^2}}{\sqrt{1+x^2}\mp\sqrt{1-x^2}}$ | $x^2 = \cos2\theta$ |

**Proof of some important properties and formulas of ITF :**

**3.** Let $\sin^{-1}(-x) = \theta$

$\Rightarrow \sin\theta = -x$ [by definition of sine inverse]

$\Rightarrow -\sin\theta = x$

$\Rightarrow \sin(-\theta) = x$ [using $\sin(-\theta) = -\sin\theta$]

$\Rightarrow \sin^{-1}x = -\theta$ [by definition of sine inverse]

$\Rightarrow -\sin^{-1}x = \theta$

$\Rightarrow \sin^{-1}(-x) = -\sin^{-1}x$

Let $\cos^{-1}(-\mathrm{x}) = \theta$

$\Rightarrow \cos\theta = -x$ [by definition of cosine inverse]

$\Rightarrow -\cos\theta = x$

$\Rightarrow \cos(\pi - \theta) = x$ [using $\cos(\pi - \theta) = -\cos\theta$]

$\Rightarrow \cos^{-1}x = (\pi - \theta)$ [by definition of cosine inverse]

$\Rightarrow \theta = \pi - \cos^{-1}x$

$\Rightarrow \cos^{-1}(-\mathrm{x}) = \pi - \cos^{-1}x$

4. Let $\sin^{-1}\frac{1}{x} = \theta$

$\Rightarrow \sin\theta = \frac{1}{x}$ [by definition of sine inverse]

$\Rightarrow \mathrm{cosec}\,\theta = x$

$\Rightarrow \theta = \mathrm{cosec}^{-1}x$ [by definition of sine inverse]

$\Rightarrow \sin^{-1}\frac{1}{x} = \mathrm{cosec}^{-1}x$

5. $\boldsymbol{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$

Let $\sin^{-1}x = \theta$

$\Rightarrow \sin\theta = x$ [by definition of sine inverse]

$\Rightarrow \cos(\frac{\pi}{2} - \theta) = x$ [$\because \sin\theta$ *bijective in* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$]

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \theta$ [by definition of cosine inverse]

$\Rightarrow \theta + \cos^{-1}x = \frac{\pi}{2}$

$\boldsymbol{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$

**6.** $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$ , *where* $x > 0, y > 0, xy < 1$

**Let** $\tan^{-1}x = A$ *and* $\tan^{-1}y = B$

$\Rightarrow \tan A = x \ \ and \ \ \tan B = y$

$\therefore LHS = A + B$

$RHS = \tan^{-1} \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \tan^{-1} \tan(A + B)$

$= A + B$

$\therefore \tan^{-1} x + \tan^{-1} y = \tan^{-1} \dfrac{x + y}{1 - xy}$

8. $\tan^{-1} x + \tan^{-1} y \ + \tan^{-1} z = \tan^{-1} \left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

**Let** $\tan^{-1} x = A$ $, \tan^{-1} y = B \ \ and \ \ \tan^{-1} z = C$

$\Rightarrow \tan A = x$ $, \tan B = y \ \ and \ \ \tan C = Z$

$\therefore LHS = A + B + C$

$RHS = \tan^{-1} \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$= \tan^{-1} \tan(A + B + C)$

$= A + B + C$

$\therefore \tan^{-1} x + \tan^{-1} y \ + \tan^{-1} z = \tan^{-1} \left(\dfrac{x + y + z - xyz}{1 - xy - yz - zx}\right)$

9. $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$ ,where $-\frac{1}{2} \leq x \leq \frac{1}{2}$

Let $\sin^{-1} x = A \Rightarrow \sin A = x$

$\therefore LHS = 3A$

$RHS = \sin^{-1}(3x - 4x^3)$

$= \sin^{-1}(3 \sin A - 4 \sin^3 A) \quad = \sin^{-1} \sin 3A \quad = 3A$

$\therefore 3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$

[$-\frac{\pi}{2} \leq 3\sin^{-1} x \leq \frac{\pi}{2}$

$\Rightarrow -\dfrac{\pi}{6} \leq \sin^{-1} x \leq \dfrac{\pi}{6}$

$\Rightarrow \sin\left(-\dfrac{\pi}{6}\right) \leq x \leq \sin\dfrac{\pi}{6}$

$\Rightarrow -\dfrac{1}{2} \leq x \leq \dfrac{1}{2}$]

10 $3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x)$ ,where $\frac{1}{2} \leq x \leq 1$

Let $\cos^{-1} x = A \Rightarrow \cos A = x$

$\therefore LHS = 3A$

$RHS = \cos^{-1}(4x^3 - 3x)$

$$= \cos^{-1}(4\cos^3 A - 3\cos A) = \cos^{-1}\cos 3A = 3A$$

$\therefore 3\,\mathrm{COS}^{-1} x = \mathrm{COS}^{-1}(4x^3 - 3x)$

13. $2\tan^{-1} x = \begin{cases} \sin^{-1}\dfrac{2x}{1+x^2} \\ \cos^{-1}\dfrac{1-x^2}{1+x^2} \\ \tan^{-1}\dfrac{2x}{1-x^2} \end{cases}$

Let $\tan^{-1} x = A \Rightarrow \tan A = x$

Now $\sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}\frac{2\tan A}{1+\tan^2 A} = \sin^{-1}\sin 2A = 2A$

$$\cos^{-1}\frac{1-x^2}{1+x^2} = \cos^{-1}\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^{-1}\cos 2A = 2A$$

$$\tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{2\tan A}{1-\tan^2 A} = \tan^{-1}\tan 2A = 2A$$

$\therefore 2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$

14. $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$ ,where $x, y \in [-1,1]$

Let $\sin^{-1} x = A \Rightarrow \sin A = x$

$\sin^{-1} y = B \Rightarrow \sin B = y$

$\therefore LHS = A + B$

$$RHS = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$= \sin^{-1}\left(\sin A\sqrt{1-\sin^2 B} + \sin B\sqrt{1-\sin^2 A}\right) = \sin^{-1}(\sin A\cos B + \cos A\sin B)$$

$$= \sin^{-1}\sin(A+B)$$

$$= A + B$$

$\therefore \sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

16. $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

Let $\cos^{-1} x = A \Rightarrow \cos A = x$

$\cos^{-1} y = B \Rightarrow \cos B = y$

$\therefore LHS = A + B$

$$RHS = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$= \cos^{-1}\left(\cos A\cos B - \sqrt{1-\sin^2 A}\sqrt{1-\sin^2 B}\right) = \cos^{-1}(\cos A\cos B - \sin A\sin B)$$

$$= \cos^{-1}\cos(A+B)$$

$$= A + B$$

$\therefore \cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

**Deduction:**

$\mathbf{3}\tan^{-1} x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$ , where $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

**Let** $\boldsymbol{\tan^{-1} x = A} \Rightarrow \tan A = x$

$\therefore LHS = 3A$

$RHS = \tan^{-1}\frac{3x-x^3}{1-3x^2} = \tan^{-1}\frac{3\tan A - \tan^3 A}{1-3\tan^2 A} = \tan^{-1}\tan 3A = 3A$

$\therefore\ \mathbf{3}\tan^{-1} x = \tan^{-1}\dfrac{3x-x^3}{1-3x^2}$

$\left\{-\dfrac{\pi}{2} < 3\tan^{-1} x < \dfrac{\pi}{2} \Rightarrow -\dfrac{\pi}{6} < \tan^{-1} x < \dfrac{\pi}{6} \Rightarrow -\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}}\right\}$

$2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}(1-2x^2)$

Let $\sin^{-1} x = A \Rightarrow \sin A = x$

Now $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}\left(2\sin A\sqrt{1-\sin^2 A}\right)$

$= \sin^{-1}(2\sin A\cos A)$

$= \sin^{-1}\sin 2A$

$= 2A$

Again $\cos^{-1}(1-2x^2)$

$= \cos^{-1}(1-2\sin^2 A)$

$= \cos^{-1}\cos 2A$

$= 2A$

$\therefore\ 2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}(1-2x^2)$

Example: Find the principal values of the following:

(i) $\sin^{-1}\left(-\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (iii $\tan^{-1}(-\sqrt{3})$ (iv) )$\cos^{-1}\left(-\frac{1}{2}\right)$ (v)$\mathrm{cosec}^{-1}(-\sqrt{2})$

Sol: (i) Let $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$

$\boldsymbol{[\sin\theta = -\frac{1}{2} \Rightarrow}$
$\boldsymbol{\theta in\ iii \& iv\ quad(two\ values\ but\ \theta =}$
$\boldsymbol{\sin^{-1}\left(-\frac{1}{2}\right) \Rightarrow}$
$\boldsymbol{\theta in\ iv\ quad(unique\ value) i.e. if\ input\ x\ in\ \sin^{-1} x\ is +}$
$\boldsymbol{ve\ we\ take\ angle\ from\ \left[0, \frac{\pi}{2}\right], otherwise\ we\ take\ from\ \left[-\frac{\pi}{2}, 0\right]}$

$\Rightarrow \sin\theta = -\dfrac{1}{2} = -\sin\dfrac{\pi}{6} = \sin\left(-\dfrac{\pi}{6}\right)$

$$\Rightarrow \theta = -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Sol: (iii) Let $\tan^{-1}(-\sqrt{3}) = \theta$

$$\Rightarrow \tan\theta = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$$

$$\Rightarrow \theta = -\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Sol: (iv) Let $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$

$$\Rightarrow \cos\theta = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{2\pi}{3} \in [0, \pi]$$

Example: Find the value of the following:

(i) $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

(ii) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

(iii) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

Sol: (i) $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

Sol: (ii) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2\frac{\pi}{6} = 2\frac{\pi}{3}$

Sol: (iii) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$$= \frac{\pi}{3} - (\pi - \sec^{-1} 2) = \frac{\pi}{3} - \pi + \sec^{-1} 2 = -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$$

Example: Prove the following:

(i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

(ii) $2\tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$

(iii) $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

(iv) $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

(v) $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

(vi) $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

(vii) $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

(viii) $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

(ix) $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ , $x\in[0,1]$

(x) $\cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right)=\frac{x}{2}$ , $x\in\left(0,\frac{\pi}{4}\right)$

(xi) $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{x}{4}-\frac{1}{2}\cos^{-1}x$ , $-\frac{1}{2}\leq x\leq 1$

(xii) $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

(xiii) $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

(xiv) $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

(xv) $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$

(xvi) $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

(xvii)If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$,then $x^2+y^2+z^2+2xyz=1$

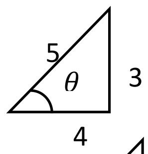
(xviii) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)=\frac{1}{2}\cos^{-1}\left(\frac{1+2\cos x}{2+\cos x}\right)$

Proof:(ii) LHS=$2\tan^{-1}\frac{1}{2}+\tan^{-1}\left(\frac{1}{7}\right)$

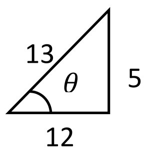
$$=\tan^{-1}\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}+\tan^{-1}\frac{1}{7}$$

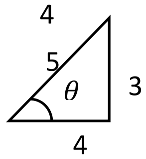
$$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}=\tan^{-1}\frac{31}{17}$$=RHS

Proof:(iii) $LHS=2\sin^{-1}\frac{3}{5}=2\tan^{-1}\frac{3}{4}=\tan^{-1}\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}=\tan^{-1}\frac{24}{7}=RHS$




Proof:(iv) $LHS=\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}$

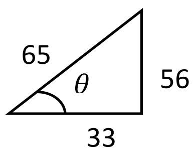



$$=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}=\tan^{-1}\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\times\frac{3}{4}}$$

$$=\tan^{-1}\frac{56}{33}$$

$$=\sin^{-1}\frac{56}{65}=RHS$$




Proof:(viii) $LHS=\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}$

$$=\tan^{-1}\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}+\tan^{-1}\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}$$

$$=\tan^{-1}\frac{12}{34}+\tan^{-1}\frac{11}{23}$$

$$= \tan^{-1} \frac{\frac{12}{34} + \frac{11}{23}}{1 - \frac{12}{34} \times \frac{11}{23}}$$

$$= \tan^{-1} \frac{12 \times 23 + 11 \times 34}{34 \times 23 - 12 \times 11} = \tan^{-1} \frac{650}{650} = \tan^{-1} 1 = \frac{\pi}{4}$$

Proof:(ix) $Let\ \tan^{-1} \sqrt{x} = \theta \Rightarrow \sqrt{x} = \tan\theta \Rightarrow x = \tan^2\theta$

RHS=$\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1}{2}\cos^{-1}\cos 2\theta = \frac{1}{2} \times 2\theta = \theta$

$= \tan^{-1}\sqrt{x}$=LHS

Proof:(x)LHS= $\cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right)$

$$= \cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}} \times \frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}+\sqrt{1-sinx}}\right)$$

$$= \cot^{-1} \frac{\left(\sqrt{1+sinx}+\sqrt{1-sinx}\right)^2}{(1+\sin x)-(1-\sin x)}$$

$$= \cot^{-1} \frac{1+\sin x+1-\sin x+2\sqrt{1-\sin^2 x}}{2\sin x}$$

$= \cot^{-1}\frac{2+2\cos x}{2\sin x}$ $\quad$ $[\because x \in \left(0, \frac{\pi}{4}\right) \Rightarrow \cos x > 0 \Rightarrow |\cos x| = \cos x]$

$$= \cot^{-1} \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \cot^{-1} \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \cot^{-1}\cot x = x = RHS$$

Proof:(xi) $LHS = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ $\quad$ let$x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1} \frac{\sqrt{2}(\cos\theta - \sin\theta)}{\sqrt{2}(\cos\theta + \sin\theta)}$$

$$= \tan^{-1} \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x = RHS$$

Proof:(xiii)LHS= $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$ $let \frac{1}{2}\sin^{-1}\frac{3}{4} = \theta \Rightarrow \sin^{-1}\frac{3}{4} = 2\theta$

$= \tan\theta$ $\Rightarrow \sin 2\theta = \frac{3}{4} \Rightarrow \cos 2\theta = \frac{\sqrt{7}}{4}$

$= \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}} = \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}} = \frac{4-\sqrt{7}}{3}$=RHS

Proof:(xviii) LHS=) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)$

$$= \frac{1}{2}\cos^{-1}\frac{1-\frac{1}{3}\tan^2\frac{x}{2}}{1+\frac{1}{3}\tan^2\frac{x}{2}}$$

$$= \frac{1}{2}\cos^{-1}\frac{3-\tan^2\frac{x}{2}}{3+\tan^2\frac{x}{2}}$$

$$= \frac{1}{2}\cos^{-1}\frac{3\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{3\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}$$

$$= \frac{1}{2}\cos^{-1}\frac{2\cos^2\frac{x}{2}+\cos x}{2\cos^2\frac{x}{2}+1}$$

$$= \frac{1}{2}\cos^{-1}\frac{1+2\cos x}{2+\cos x} = RHS$$

Example: Write the following functions in its simplest form.

(i) $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ $, x \neq 0$ (ii) ) $\tan^{-1}\frac{1}{\sqrt{x^2-1}}$ $, |x| > 0$

(iii) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ $, 0 < x < \pi$ (iv) $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$ $, 0 < x < \pi$

(v) $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$ $, |x| < a$ (vi) $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ $, a > 0$

Sol(i) $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ $let\ x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$= \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$$

$$= \tan^{-1}\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}$$

$$= \tan^{-1}\frac{\sec\theta-1}{\tan\theta}$$

$$= \tan^{-1}\frac{1-\cos\theta}{\sin\theta}$$

$= \tan^{-1} \dfrac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \tan^{-1} \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \tan^{-1} \tan\dfrac{\theta}{2}$

$= \dfrac{\theta}{2} = \dfrac{1}{2}\tan^{-1} x$

Sol(ii) $\tan^{-1}\frac{1}{\sqrt{x^2-1}}$ $\quad$ $let\ x = \sec\theta \Rightarrow \theta = \sec^{-1} x$

$= \tan^{-1}\dfrac{1}{\sqrt{\sec^2\theta - 1}}$

$= \tan^{-1}\dfrac{1}{\sqrt{\tan^2\theta}}$

$= \tan^{-1}\dfrac{1}{\tan\theta}$

$= \tan^{-1}\cot\theta$

$= \tan^{-1}\tan\left(\dfrac{\pi}{2} - \theta\right)$

$= \dfrac{\pi}{2} - \theta = \dfrac{\pi}{2} - \sec^{-1} x$

Sol(iii) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

$= \tan^{-1}\sqrt{\tan^2\dfrac{x}{2}}$

$= \tan^{-1}\tan\frac{x}{2}$ $\quad$ $[0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2} \Rightarrow \tan\frac{x}{2} > 0 \Rightarrow \left|\tan\frac{x}{2}\right| = \tan\frac{x}{2}]$

$= \dfrac{x}{2}$

Sol(iv) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\tan\left(\frac{\pi}{4} - x\right) = \frac{\pi}{4} - x$

Sol(vi) $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = \tan^{-1}\frac{3\frac{x}{a} - \frac{x^3}{a^3}}{1 - 3\frac{x^2}{a^2}} = \tan^{-1}\frac{3\left(\frac{x}{a}\right) - \left(\frac{x}{a}\right)^3}{1 - 3\left(\frac{x}{a}\right)^2} = 3\tan^{-1}\frac{x}{a}$

Example: Find the value of each of the following :

(i) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ $\quad$ (ii) $\cot(\tan^{-1} a + \cot^{-1} a)$

(iii) $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1-x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$ (iv) $\cos^{-1}\cos\left(\frac{13\pi}{6}\right)$

(iv) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ (v) $\sin\left[\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)\right]$ (vi) $\tan\left[\frac{1}{2}\cos^{-1}\frac{\sqrt{2}}{3}\right]$

(vii) $\cos\left[\frac{1}{2}\cos^{-1}\frac{1}{8}\right]$ (viii) $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

(ix) $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)$

(x) $\tan^{-1}\frac{1}{2}+\sin^{-1}\frac{1}{\sqrt{10}}$ (xi) $\cos 2\left(\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}\right)$ (xii) $\sin^{-1}\frac{4}{5}+\sec^{-1}\frac{5}{4}-\frac{\pi}{2}$

Sol: (i) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$=\tan^{-1}\left[2\times\frac{1}{2}\right]=\tan^{-1}1=\frac{\pi}{4}$

Sol: (iii) $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1-x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$=\tan\frac{1}{2}[2\tan^{-1}x+2\tan^{-1}y]$

$=\tan\frac{1}{2}\left[2\left(\tan^{-1}\frac{x+y}{1-xy}\right)\right]$

$=\tan\tan^{-1}\frac{x+y}{1-xy}=\frac{x+y}{1-xy}$

Sol: (iv) $\cos^{-1}\cos\left(\frac{13\pi}{6}\right)=\cos^{-1}\cos\left(2\pi+\frac{\pi}{6}\right)=\cos^{-1}\cos\frac{\pi}{6}=\frac{\pi}{6}$

Sol: (v) $\sin\left[\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)\right]=\sin\left(-\operatorname{cosec}^{-1}\frac{2}{\sqrt{3}}\right)=\sin\left(-\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

Sol: (vii) $\cos\left[\frac{1}{2}\cos^{-1}\frac{1}{8}\right]$

$=\cos\theta$ $\left[let\ \frac{1}{2}\cos^{-1}\frac{1}{8}=\theta\Rightarrow\cos 2\theta=\frac{1}{8}\right]$

$=\sqrt{\frac{1+\cos 2\theta}{2}}$

$=\sqrt{\frac{1+\frac{1}{8}}{2}}=\frac{3}{4}$

Sol: (viii) $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

$=\sin 2\theta$ $\left[\ let\ \cos^{-1}\left(-\frac{3}{5}\right)=\theta\Rightarrow\cos\theta=-\frac{3}{5}\right]$

$=2\sin\theta\cos\theta$

$$= 2\sqrt{1-\frac{9}{25}}\left(-\frac{3}{5}\right) = -2\times\frac{4}{5}\times\frac{3}{5} = -\frac{24}{25}$$

Sol:(ix) $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$

$= 1 + \tan^2(\tan^{-1} 2) + 1 + \cot^2(\cot^{-1} 3) = 1 + 2^2 + 1 + 3^2 = 15$

Example: Solve the following:

(i) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

(ii) $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x \quad, x > 0$

(iii) $\tan^{-1}\frac{2-x}{2+x} = \frac{1}{2}\tan^{-1}\frac{x}{2} \quad, x > 0$

(iv) $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$

(v) $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

(vi) $\tan^{-1}\frac{x-2}{x-3} + \tan^{-1}\frac{x+2}{x+3} = \frac{\pi}{4} \quad, |x| < 1$

(vii) $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

Sol: (i) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = 2\operatorname{cosec} x$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$$

$$\Rightarrow \cot x = 1$$

$$\Rightarrow x = \frac{\pi}{4}$$

Sol: (iii) $\tan^{-1}\frac{2-x}{2+x} = \frac{1}{2}\tan^{-1}\frac{x}{2}$

$$\Rightarrow \tan^{-1}\frac{1-\frac{x}{2}}{1+\frac{x}{2}} = \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$\Rightarrow \frac{\pi}{4} - \tan^{-1}\frac{x}{2} = \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$\Rightarrow \frac{3}{2}\tan^{-1}\frac{x}{2} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{x}{2} = \frac{\pi}{6}$$

$$\Rightarrow \frac{x}{2} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$\Rightarrow x = \frac{2}{\sqrt{3}}$

Sol: (iv) $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\frac{x-3}{x-4}\times\frac{x+3}{x+4}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{(x-3)(x+4)+(x+3)(x-4)}{(x-4)(x+4)-(x-3)(x+3)} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{x^2+x-12+x^2-x-12}{(x^2-16)-(x^2-9)} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{2x^2-24}{-7} = \frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-24}{-7} = \tan\frac{\pi}{4} = 1$$

$\Rightarrow 2x^2 - 24 = -7$

$\Rightarrow 2x^2 = 17$

$\Rightarrow x^2 = \frac{17}{2}$

$\Rightarrow x = \pm\sqrt{\frac{17}{2}}$

Sol: (v) $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{2x+3x}{1-6x^2} = \frac{\pi}{4}$$

$$\Rightarrow \frac{2x+3x}{1-6x^2} = \tan\frac{\pi}{4} = 1$$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow 6x(x+1) - (x+1) = 0$

$\Rightarrow (x+1)(6x-1) = 0$

$\Rightarrow x = -1, \frac{1}{6}$

But $x = -1$ does not satisfies the equation .

Requires solution is $x = \frac{1}{6}$.

Sol: (vi) $\tan^{-1}\frac{x-2}{x-3} + \tan^{-1}\frac{x+2}{x+3} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{\frac{x-2}{x-3}+\frac{x+2}{x+3}}{1-\frac{x-2}{x-3}\times\frac{x+2}{x+3}}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{(x-2)(x+3)+(x+2)(x-3)}{(x-3)(x+3)-(x-2)(x+2)}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{x^2+x-6+x^2-x-6}{(x^2-9)-(x^2-4)}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{2x^2-12}{-5}=\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-12}{-5}=\tan\frac{\pi}{4}=1$$

$$\Rightarrow 2x^2-12=-5$$

$$\Rightarrow 2x^2=7$$

$$\Rightarrow x^2=\frac{7}{2}$$

$$\Rightarrow x=\pm\sqrt{\frac{7}{2}}\in -1<x<1$$

The equation has no solution.

Sol: (vii) $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$

$$\Rightarrow \sin^{-1}x+\sin^{-1}2x=\sin^{-1}\frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin^{-1}x-\sin^{-1}\frac{\sqrt{3}}{2}=-\sin^{-1}2x$$

$$\Rightarrow \sin^{-1}\left\{x\sqrt{1-\frac{3}{4}}-\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right\}=\sin^{-1}(-2x)$$

$$\Rightarrow \frac{x}{2}-\frac{\sqrt{3}}{2}\sqrt{1-x^2}=-2x$$

$$\Rightarrow \frac{x}{2}+2x=\frac{\sqrt{3}}{2}\sqrt{1-x^2}$$

$$\Rightarrow \frac{5x}{2}=\frac{\sqrt{3}}{2}\sqrt{1-x^2}$$

$$\Rightarrow 25x^2=3(1-x^2)$$

$$\Rightarrow 28x^2=3$$

$$\Rightarrow x^2=\frac{3}{28}$$

$$\Rightarrow x = \pm\sqrt{\frac{3}{28}} = \pm\frac{\sqrt{3}}{2\sqrt{7}}$$

But x can't be negative ,

The required solution is $x = \frac{\sqrt{3}}{2\sqrt{7}}$

**Example:** Let $x, y, z$ be positive real numbers such that $x, y, z$ $are$ in G.P. and $\tan^{-1} x$, $\tan^{-1} y$ , $\tan^{-1} z$ are in A.P. ,then which of the following is correct ?(2017-I,JEE-2013)

(a) $x = y = z$ (b) $x.z = 1$ (c) $x \neq y$ $and$ $y = z$ (d) ) $x = y$ $and$ $y \neq z$

Sol: $2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z = \tan^{-1}\frac{x+z}{1-xz}$

$$\Rightarrow \tan^{-1}\frac{2y}{1-y^2} = \tan^{-1}\frac{x+z}{1-xz}$$

$$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-y^2}$ $[\because xz = y^2]$

$\Rightarrow 2y = x + z$

**Example:** What is value of $\cos(2\cos^{-1} 0.8)$?

(a) 0.81 (b) 0.56 (c) 0.48 (d) 0.28

Sol: Let $\cos^{-1} 0.8 = \theta \Rightarrow \cos\theta = 0.8 \Rightarrow \cos 2\theta = 2\cos^2\theta - 1 = 2 \times 0.64 - 1 = 1.28 - 1 = 0.28$

**Ans (d)**

**Example:** The value of $\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n} 2p\right)\right]$

(a) $\frac{21}{19}$ (b) $\frac{19}{4}$ (c) $\frac{20}{23}$ (d) $\frac{23}{22}$

Sol:Ans (a) $\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n} 2p\right)\right] = \cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+2\sum_{p=1}^{n} p\right)\right]$

$$= \cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+2\frac{n(n+1)}{2}\right)\right]$$

$$= \cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+n(n+1)\right)\right]$$

$$= \cot\left[\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right] = \cot\left[\sum_{n=1}^{19}\tan^{-1}\frac{(n+1)-n}{1+n(n+1)}\right]$$

$$= \cot\left[\sum_{n=1}^{19}\tan^{-1}(n+1) - \tan^{-1} n\right] = \cot[\tan^{-1} 20 - \tan^{-1} 1] = \cot\tan^{-1}\frac{19}{21} = \cot\cot^{-1}\frac{21}{19} = \frac{21}{19}$$

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## Practice questions

## Section-I (1 mark each)

**MCQ type**

1.$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{3}$

Ans: (b)

2. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) $[1,2]$ (b) $(1,2)$ (c) $[1,2)$ (d) $(1,2]$

Ans: (a)

3. The principal values of $\tan^{-1}(-\sqrt{3})$ and $\cos^{-1}\left(-\frac{1}{2}\right)$ are

(a) $-\frac{\pi}{3}$ and $-\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ and $\frac{2\pi}{3}$ (c) $-\frac{\pi}{3}$ and $\frac{2\pi}{3}$ (d) $\frac{2\pi}{3}$ and $-\frac{4\pi}{3}$

Ans: (c)

4. $\cot^{-1}(-\sqrt{3}) + \tan^{-1}(1)$ is equal to

(a) $\frac{\pi}{12}$ (b) $\frac{13\pi}{12}$ (c) $\frac{13\pi}{2}$ (d) $\frac{\pi}{2}$

Ans: (b)

5. $\tan^{-1}\left(\sqrt{2}\sin\frac{3\pi}{4}\right)$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Ans: (d)

6. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$,then $(\alpha^3 + \beta^3 + \gamma^3) - \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$ is

(a) $1$ (b) $0$ (c) $-1$ (d) $2$

Ans: (b)

7. $\cot\left(\frac{\pi}{3} - 2\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$ is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $1$ (d) $-\frac{1}{\sqrt{3}}$

Ans: (d)

8. If $x = \cos^{-1}\left(-\frac{1}{2}\right)$,then $x + \frac{\pi}{3}$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\pi$ (c) $2\pi$ (d) $3\pi$

Ans: (b)

9.The domain of $f(x) = \sin^{-1}(-x^2)$ is

(a) $(-1,1)$ (b) $[-1,1]$ (c) $(-1,1]$ (d) $[-1,1)$

Ans: (b)

Sol: The domain of $\sin^{-1}x$ is $[-1,1]$

$\Rightarrow -1 \le -x^2 \le 1$

$\Rightarrow 1 \ge x^2 \ge -1$

$\Rightarrow 0 \le x^2 \le 1$

$\Rightarrow x^2 \le 1$

$\Rightarrow -1 \le x \le 1$

10. The principal values of $\sec^{-1}(-\sqrt{2})$ +$\text{cosec}^{-1}(-\sqrt{2})$ is

a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-\pi}{2}$

Ans: (a)

11.The value of $\cos^{-1}\left(\sin\frac{\pi}{6}\right)$ is

a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Ans: (b)

12. Which of the following is the principal value branch of $\cos^{-1}x$ ?

(a) $[-\frac{\pi}{2},\frac{\pi}{2}]$ (b) $(0,\pi)$ (c) $[0,\pi]$ (d) $(0,\pi)-\left\{\frac{\pi}{2}\right\}$

**Assertion –Reason Based Questions**

In the following questions ,a statement of Assertion (A) is followed by a statement of Reason (R) Choose the correct answer out of the following choices

(a) Both A and R true and R is the correct explanation of A .

(b) Both A and R true and R is not the correct explanation of A .

(c ) A is true but R is false .

(d)A is false but R is true .

1. Assertion (A) The domain of the function $\sec^{-1}2x$ is $(-\infty,-\frac{1}{2}]\cup[\frac{1}{2},\infty)$

Reason(R) $\sec^{-1}(-2)=-\frac{\pi}{4}$

Ans: (c)

Since $\sec^{-1}x$ *is defined* , *if* $x \le -1$ *or* $x \ge 1$

Hence $\sec^{-1}2x$ *will be defined* , *if* $x \le -\frac{1}{2}$ *or* $x \ge \frac{1}{2}$

Hence A is true.

The range of the function $\sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

R is false.

## Section-II(2 mark each)

1. Solve for $x$: $tan^{-1}2x + tan^{-1}3x = \frac{\pi}{4}$

2. Find $x$ $if sin^{-1}\frac{3}{5} + tan^{-1}\frac{3}{4} = tan^{-1}x$ .

3. P.T $tan^{-1}\left(\frac{\sqrt{1+cosx}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}\right) = \frac{\pi}{4} - \frac{x}{2}$ , $\pi < x < \frac{3\pi}{2}$

4. Simplify $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$ ,if $\frac{a}{b}\tan x > -1$

5. Show that $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

6. Solve for $x$: $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$.

7. Prove the following: $\cos[\tan^{-1}\{sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

8. Express $\sin^{-1}\left(\frac{sinx+siny}{\sqrt{2}}\right)$ where $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ , in the simplest form

9. Solve : $\tan^{-1} 4x + \tan^{-1} 6x = \pi/4$

10. Find the number of solutions of the equation : $2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$ $if$ $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

**Sol:** Given equation is $2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x + (\sin^{-1}x + \cos^{-1}x) = \frac{11\pi}{6}$.

$\Rightarrow \cos^{-1}x + \frac{\pi}{2} = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{11\pi}{6} - \frac{\pi}{2} = \frac{4\pi}{3}$

Which is not possible as $\cos^{-1}x \in [0, \pi]$

Thus, given equation has no solution.

12. Find the domain of the function $f(x) = \cos^{-1}x + \sin^{-1}2x$

Ans: $[-\frac{1}{2}, \frac{1}{2}]$

13. Find the value of $\sin^{-1}(\sin\frac{13\pi}{7})$

Ans: $-\frac{\pi}{7}$

## Section-III(3 mark each)

1.Prove that: $tan^{-1}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\pi}{4} - \frac{1}{2}cos^{-1}x$, $-\frac{1}{\sqrt{2}} \le x \le 1$

2. If $tan^{-1}\frac{x-2}{x-4} + tan^{-1}\frac{x+2}{x+4} = \frac{\pi}{4}$, find the value of $x$

3. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$ , $x \in \left(0, \frac{\pi}{4}\right)$

4. Solve : $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

13. Prove that : $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

14. Prove the following: $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

15. Prove the following:- $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

**Section-IV(4 mark each)**

1. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

2. Show that $2\sin^{-1}\frac{3}{4}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

3.Solve the following equation:- $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$

4.Find the value of :$sin\left(2tan^{-1}\frac{1}{4}\right)+\ cos\left(tan^{-1}2\sqrt{2}\right)$

5. $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$

6.If $\tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}=\alpha$,prove that $x^2=\sin 2\alpha$

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CHAPTER-3 MATRICES

**Application of Matrices**

Used to check the consistency of system of linear equations of two or more variables and to find their solutions.

Used to make spreadsheet programs for computer.

Used in computer graphics to manipulate 3D models and project them onto a 2D screen.

Used in different branches of engineering, social science, economics etc.

## Contents


1. Definition of matrix
2. Types of Matrices with examples
3. Transpose of a matrix with examples
4. Symmetric and skew-symmetric matrices with examples
5. Singular and non-singular matrices with examples
6. Equality of matrices with examples
7. Addition of matrices with properties
8. Multiplication of matrices by a scalar

9.Multiplication of two matrices with properties

10.Adjoint of a square matrix with properties

11.Inverse of a square matrix with properties

12.Existence of non-zero matrices having product zero matrix

13.Uniquenes of inverse

14.Matrix Polynomial

15.Solution of a system of linear equations by matrix method

16.Consistency of a system of linear equations


**Matrix:** A rectangular arrangement of $mn$ numbers (or elements)in $m$ rows and $n$ columns and enclosed within a pair of first bracket ( ) or a pair of third bracket [ ] is called a matrix of order $m \times n$ (read as $m$ by $n$).

Thus $m \times n$ matrix $A$ is written as

$$\begin{pmatrix} a_{11} & a_{12} & \cdots\cdots\cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots\cdots\cdots & a_{2n} \\ \cdots & \cdots & \cdots\cdots\cdots\cdots\cdots & \\ \cdots & \cdots & \cdots\cdots\cdots\cdots\cdots & \\ a_{m1} & a_{m2} & \cdots\cdots\cdots\cdots & a_{mn} \end{pmatrix} \begin{matrix} \longrightarrow R_1 \\ \longrightarrow R_2 \\ \\ \\ \longrightarrow R_m \end{matrix}$$

$$\downarrow C_1 \quad \downarrow C_2 \qquad\qquad \downarrow C_n$$

Or in short $A = (a_{ij})_{m\times n}$ ,where $a_{ij}$ belongs to $i^{th}$ row and $j^{th}$ column .

**Note:** A determinant is a single number ,called its value where as a matrix is a system of numbers and has no value.

## Types of Matrices:

1.**Row matrix:** A matrix having only one row is called a row matrix.

$$\text{Example} \rightarrow (a_{11} \;\; a_{12} \;\; a_{13} \;\; a_{14})_{1\times 4}$$

2. **Column matrix:** A matrix having only one column is called a column matrix.

$$\text{Example} \rightarrow \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \\ a_{51} \end{pmatrix}_{5\times 1}$$

3.**Rectangular matrix:** A matrix in which number of rows are not equal to the number of columns is called a rectangular matrix.

Example$\rightarrow \begin{pmatrix} 1 & 2 \\ 0 & -3 \\ -4 & 7 \end{pmatrix}_{3\times 2}$

4. **Square matrix:** A matrix in which number of rows are equal to the number of columns is called a square matrix.

Example$\rightarrow \begin{pmatrix} 2 & 5 & 0 \\ 3 & 1 & -4 \\ 7 & 5 & 12 \end{pmatrix}_{3\times 3}$

5.**Diagonal matrix:** A square matrix whose non-diagonal elements are all zeros is called a diagonal matrix.

Example→ $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}_{3\times3}$

6. **Scalar matrix:** A diagonal matrix whose all diagonal elements are equal is called a scalar matrix.

Example→ $\begin{pmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{pmatrix}_{3\times3}$

7. **Unit matrix(or Identity matrix):** A square matrix whose diagonal elements are all unity (i.e 1) and non-diagonal elements are all zeros is called a unit matrix.

Example→ $\mathrm{I}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3}$ $\mathrm{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times2}$

8. **Null matrix(or zero matrix):** A matrix of any order is said to be null matrix if all its elements are zeros.

Example→ $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times2}$ ,$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}_{3\times2}$

9.**Upper triangular matrix:** A square matrix whose all elements below the principal diagonal are zeros is called upper triangular matrix.

Example→ $\begin{pmatrix} 1 & 3 & -2 \\ 0 & 7 & 4 \\ 0 & 0 & 5 \end{pmatrix}_{3\times3}$ $i.e\ a_{ij} = 0 \ \ ,\forall\ i > j$

10. **Lower triangular matrix:** A square matrix whose all elements above the principal diagonal are zeros is called upper triangular matrix.

Example→ $\begin{pmatrix} 1 & 0 & 0 \\ 4 & 7 & 0 \\ 3 & -1 & 5 \end{pmatrix}_{3\times3}$ $i.e\ a_{ij} = 0 \ \ ,\forall\ i < j$

11. **Transpose of a matrix :** If A is a matrix of any order ,then the transpose of A is obtained by interchanging its rows and columns and is denoted by A' or $A^T$

Example→ $A = \begin{pmatrix} 1 & 3 \\ -2 & 7 \end{pmatrix}_{2\times 2} \Rightarrow A^T = \begin{pmatrix} 1 & -2 \\ 3 & 7 \end{pmatrix}_{2\times 2}$

$$A = \begin{pmatrix} 3 & 4 \\ 2 & -5 \\ 7 & 1 \end{pmatrix}_{3\times 2} \Rightarrow A^T = \begin{pmatrix} 3 & 2 & 7 \\ 4 & -5 & 1 \end{pmatrix}_{2\times 3}$$

**Properties of Transpose :**

(i) $(A^T)^T = A$

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(AB)^T = B^T A^T$ [transpose of products=product of transposes taken in reverse order]

(iv) $(A^n)^T = (A^T)^n$

(v) $(\lambda A)^T = \lambda A^T$ ,where $\lambda \in R$

(vi) $|A^T| = |A|$

Example: For $= \begin{pmatrix} 1 & 3 \\ -2 & 7 \end{pmatrix}$, $A^T = \begin{pmatrix} 1 & -2 \\ 3 & 7 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 3 \\ -2 & 7 \end{vmatrix} = 13$ and $|A^T| = \begin{vmatrix} 1 & -2 \\ 3 & 7 \end{vmatrix} = 13$

$\therefore |A^T| = |A|$

12. **Symmetric matrix :** A square matrix A is said to be symmetric if $A^T = A$

[i.e $a_{ij} = a_{ji}$ $, \forall\ i, j$]

Example→ $A = \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}_{3\times 3}$ $is\ symmetric$

$$A = \begin{pmatrix} 2 & -3 & 4 \\ -3 & 5 & 1 \\ 4 & 1 & -7 \end{pmatrix}_{3\times 3} \quad is\ symmetric$$

13. **Skew-symmetric matrix:** A square matrix A is said to be skew-symmetric if $A^T = -A$ [i.e $a_{ij} = -a_{ji}$ $, \forall\ i, j$]

Example→ $A = \begin{pmatrix} 0 & 2 & -5 \\ -2 & 0 & 3 \\ 5 & -3 & 0 \end{pmatrix}$ $is\ skew - symmetric.$

**Note:** (i) For a skew-symmetric matrix ,diagonal elements must be zero.

(ii) If A is both symmetric and skew-symmetric matrix ,then A is a null matrix.

Proof(i): Let A be a skew-symmetric matrix.

Then $(i,j)^{th} element\ of\ A = -(j,i)^{th} element\ of\ A\ , \forall\ i,j$

i.e $a_{ij} = -a_{ji} \quad \forall\ i,j$

$\Rightarrow a_{ii} = -a_{ii}$ [putting j=i]

$\Rightarrow 2a_{ii} = 0$

$\Rightarrow a_{ii} = 0$

i.e all diagonal elements of skew-symmetric matrix are zero.

Proof(ii): Let A be both symmetric and skew-symmetric matrix.

Then $A^T = A$ $[\because A\ symmetric]$

$A^T = -A$ $[\because A\ skew - symmetric]$

$\therefore A = -A$

$\Rightarrow 2A = 0$

$\Rightarrow A = 0$ i.e A is a null matrix.

14. **Singular and non-singular matrix:** A square matrix A is said to be singular if $|A| = 0$ and non-singular if $|A| \neq 0$

Example: $A = \begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 8 \\ 1 & 4 \end{vmatrix} = 0 \Rightarrow A\ is\ singular$

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = -2$$

$$\Rightarrow |A| \neq 0$$

$$\Rightarrow A\ is\ non - singular$$

**Equality of two matrices:** Two matrices A and B are said to be equal iff

(i) they are of same order .

(ii) each element of A is equal to the corresponding element of B.

Example: Find $x \; if \begin{pmatrix} 2x-y & 5 \\ 3 & y \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 3 & -2 \end{pmatrix}$

Sol$^n$: Here $y = -2$

$2x - y = 6$

$\Rightarrow 2x + 2 = 6$

$\Rightarrow 2x = 4 \Rightarrow x = 2$

**Addition of matrices:** Two or more matrices can be added (or subtracted) if they are of same order and the elements of the resulting matrix are the sum(or subtraction) of the corresponding elements of the matrices.

Example: If $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -5 & 2 \end{pmatrix} \; and \; B = \begin{pmatrix} -2 & 4 & 0 \\ 3 & 5 & -1 \end{pmatrix}$ ,find

$A + B \; and \; B - A$.

Sol$^n$: $A + B = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -5 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 4 & 0 \\ 3 & 5 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 6 & 0 \\ 6 & 0 & 1 \end{pmatrix}$

$$B - A = \begin{pmatrix} -2 & 4 & 0 \\ 3 & 5 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 0 \\ 3 & -5 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 0 \\ 0 & 10 & -3 \end{pmatrix}$$

**Properties of matrix addition:** If A,B,C are of same order, then

(i) $A + B = B + A$ (i.e matrix addition is commutative)

(ii) $(A + B) + C = A + (B + C)$ (i.e matrix addition is associative)

(iii) $A + 0 = 0 + A = A$ (i.e null matrix is additive identity)

**Multiplication of a matrix by a scalar(i.e real number):**

If $\lambda$ is a scalar and A is a matrix of any order, then $\lambda A$ is defined as the matrix in which all the elements of A are multiplied by $\lambda$.

Example: $2\begin{pmatrix} 1 & 0 \\ 5 & -3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 10 & -6 \\ 8 & 4 \end{pmatrix}$

**Multiplication of two matrices:** Let A and B be two matrices .

Then AB is defined (or possible) if number of column in A(here pre-matrix) is equal to the number of rows in B(here post-matrix).

For example , if two matrices are $A_{m\times n}$ and $B_{n\times p}$,then AB is defined and the order of AB is $m \times p$.

But BA is not defined as number of column in B(here pre-matrix)is not equal to the number of rows in A(here post-matrix).

**Working rules for multiplication AB ,when AB is defined.**

Let the two matrices be $A_{2\times 3}$ and $B_{3\times 3}$,then to find product AB

Step1:Take the elements of first row of A and elements of first column of B and take their product element-wise.

Step2: Take the sum of all the products done in step1 and this sum will be the element of first row and first column of matrix AB.

Step3: Repeat step1 taking first row elements of A and second column elements of B and then using step2 ,we will get element of first row and second column of matrix AB, and continue the process of step1 and step2 upto third columns of B. This will give all the elements of first row of AB.

Step4: Repeat step1 and step2 with second row of A and with first column , with second and with third column of B respectively. This will give all the elements of second row of AB.

Step5: As no rows are left in A and so multiplication AB is over.



Example1:If $A = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \end{pmatrix}_{2\times 3}$ $and\ B = \begin{pmatrix} 6 & 1 \\ 2 & 3 \\ 4 & 5 \end{pmatrix}_{3\times 2}$ ,find AB and BAif possible.

Sol$^n$: Here AB is possible ,as number of column of A (pre-matrix) is equal to the number of rows of B(post-matrix).

Now $AB = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 6 & 1 \\ 2 & 3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 12+6+20 & 2+9+25 \\ 6+0+8 & 1+0+10 \end{pmatrix}$

$$= \begin{pmatrix} 38 & 36 \\ 14 & 11 \end{pmatrix}_{2\times 2}$$

Again BA is also possible ,as number of column of B (pre-matrix) is equal to the number of rows of A(post-matrix).

Now $BA = \begin{pmatrix} 6 & 1 \\ 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 12+1 & 18+0 & 30+2 \\ 4+3 & 6+0 & 10+6 \\ 8+5 & 12+0 & 20+10 \end{pmatrix}$

$$= \begin{pmatrix} 13 & 18 & 32 \\ 7 & 6 & 16 \\ 13 & 12 & 30 \end{pmatrix}$$

Example2: If $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}_{2\times 3}$ $and\ B = \begin{pmatrix} 0 & -2 \\ 3 & 1 \\ -2 & 0 \end{pmatrix}_{3\times 2}$ ,find AB and BA if possible.

Sol$^n$: Here AB is possible and

$$AB = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ 3 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 0+6-6 & -2+2+0 \\ 0-6+6 & 2-2+0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
$$= 0(null\ matrix)$$

BA is also possible and

$$BA = \begin{pmatrix} 0 & -2 \\ 3 & 1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 0+2 & 0+4 & 0+6 \\ 3-1 & 6-2 & 9-3 \\ -2+0 & -4+0 & -6+0 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 4 & 6 \\ 2 & 4 & 6 \\ -2 & -4 & -6 \end{pmatrix}$$

**Note:** Multiplication of diagonal matrices of same order is commutative.

**Properties of matrix multiplication:**

1.Matrix multiplication is not commutative in general i.e $AB \neq BA$ even if both AB and BA are defined.

2. Matrix multiplication is associative i.e $(AB)C = A(BC)$,where respective multiplications are defined.

3.Matrix multiplication is distributive over matrix addition i.e

$$\mathrm{A(B + C) = AB + AC}$$

$$(A + B)C = AC + BC$$

4. If $AB = 0$(null matrix) ,then both A and B may not be null matrix and then A and B are called divisors of zero.

5. $AI = IA = A$, where $I$ is the identity matrix.

6.$A0 = 0A = 0$ ,where 0 is the null matrix.

7 . $I^2 = I^3 = I^4 = \cdots \ldots . = I$,where I is the identity matrix.

8. $A^{n+1} = A^n \times A \, , where \; n \in N$.

9. $|AB| = |A||B|$

10. If $A$ is a square matrix of order $n$ and $\lambda$ is any scalar then

$|\lambda A| = \lambda^n |A|$.

For example ,let $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -1 & 1 \\ 3 & 4 & 5 \end{pmatrix}$ $and\ \lambda = 2$,then

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 4 & -1 & 1 \\ 3 & 4 & 5 \end{vmatrix} = 1(-5-4) + 2(20-3) + 3(16+3)$$

$$= -9 + 34 + 57 = 82$$

$$\text{Now } |2\mathrm{A}| = \begin{vmatrix} 2 & -4 & 6 \\ 8 & -2 & 2 \\ 6 & 8 & 10 \end{vmatrix} = 2 \times 2 \times 2 \begin{vmatrix} 1 & -2 & 3 \\ 4 & -1 & 1 \\ 3 & 4 & 5 \end{vmatrix} = 2^3 \times 82$$

Example: Construct a $2 \times 2$ matrix ,whose elements are given by

(i) $a_{ij} = \frac{(i+j)^2}{2}$ (ii) $a_{ij} = \frac{1}{2}|-3i + j|$

Sol$^n$: (i) Let the $2 \times 2$ matrix be $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

Now $a_{11} = \frac{(1+1)^2}{2} = 2, \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, \qquad a_{22} = \frac{(2+2)^2}{2} = 8$$

$$\therefore A = \begin{pmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{pmatrix}$$

Sol$^n$: (ii) Let the $2 \times 2$ matrix be $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

Now $a_{11} = \frac{1}{2}|-3+1| = 1, \quad a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}$

$$a_{21} = \frac{1}{2}|-6+1| = \frac{5}{2}, \qquad a_{22} = \frac{1}{2}|-6+2| = 2$$

$$\therefore A = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{pmatrix}$$

Example: If a matrix has 24 elements ,what are the possible orders it can have ? What if it has 13 elements?

Sol$^n$: The possible ordered pairs ,whose product is 24 are $(1,24), (1,24), (2,12), (12,2), (3,8), (8,3), (4,6), (6,4)$.

$\therefore$ The possible orders for a matrix to have 24 elements are 8.

Again , the possible ordered pairs ,whose product is 13 are $(1,13) and (13,1)$

$\therefore$ The possible orders for a matrix to have 13 elements are 2.

Example: Find $X$ and $Y$ if $2X + Y = \begin{pmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{pmatrix}$ and

$$X - 2Y = \begin{pmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

Sol$^n$: $2X + Y = \begin{pmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{pmatrix}$

$$\Rightarrow 4X + 2Y = 2\begin{pmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 8 & 8 & 14 \\ 14 & 6 & 8 \end{pmatrix} \rightarrow (1)$$

$$X - 2Y = \begin{pmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix} \rightarrow (2)$$

Adding (1) and (2) we get

$$5X = \begin{pmatrix} 5 & 10 & 15 \\ 15 & 5 & 10 \end{pmatrix}$$

$$\Rightarrow X = \frac{1}{5}\begin{pmatrix} 5 & 10 & 15 \\ 15 & 5 & 10 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

From (2) $\quad 2Y = X - \begin{pmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}$$

$$\Rightarrow Y = \tfrac{1}{2}\begin{pmatrix} 4 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Example: If A is a square matrix of order 3 and $|A| = 4$,find $|3A|$.

Sol$^n$: $|3A| = 3^3|A| = 27 \times 4 = \ 108.$

Example: If A and B are symmetric matrices of same order ,then $(AB - BA)$ is a

(a)Skew-symmetric matrix (b) symmetric matrix

(c) zero matrix (d) Identity matrix

Sol$^n$: $(AB - BA)^T = (AB)^T - (BA)^T [\because (A - B)^T = A^T - B^T]$

$= B^T A^T - A^T B^T \ [\because (AB)^T = B^T A^T]$

$= BA - AB \quad [\because \text{A and B are symmetric}]$

$= -(AB - BA)$

$\therefore \text{(AB} - \text{BA) is skew} - \text{symmetric matrix}$

$Ans\ (a)$

Example: If A is a skew-symmetric matrix of odd order ,then $|A| = 0$

Proof: Since A is skew-symmetric

$\therefore A^T = -A$

$\Rightarrow |A^T| = |-A| = (-1)^n|A| = -|A|$, where $n$ is order of matrix

$\Rightarrow |A| = -|A| \qquad [\because |A^T| = |A|]$

$\Rightarrow 2|A| = 0$

$\Rightarrow |A| = 0$

Example: Given a skew-symmetric matrix $A = \begin{pmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{pmatrix}$,find the value of $(a+b+c)^2$

Sol$^n$: Since A is skew-symmetric

$\therefore a_{ij} = -a_{ji} \ \forall \, i,j$ and $a_{ii} = 0$

$\Rightarrow a = 1, b = 0, c = -1$

$\therefore (a+b+c)^2 = (1+0-1)^2 = 0$

Example: $If\ A = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,find scalar $k$ so that $A^2 + I = kA$.

Sol$^n$: $A^2 + I = kA$.

$\Rightarrow \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = k\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = k\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 12 & -8 \\ -4 & 4 \end{pmatrix} = k\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$

$\Rightarrow -4\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix} = k\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$

$\Rightarrow k = -4$

**Matrix polynomial :** If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots \ldots . + a_1 x + a_0$

is a polynomial in $x$ and $A$ is a square matrix of order $n$, then

then $f(A) = a_n A^n + a_{n-1} x A^{n-1} + \cdots \ldots . + a_1 A + a_0 I$

is called a matrix polynomial.

Example: $If\ f(x) = x^2 + 2x + 3, then\ f(A) = A^2 + 2A + 3I$

**Theorem:** Every square matrix is uniquely expressible as the sum of a symmetric and a skew-symmetric matrices.

Proof: Let A be a square matrix.

Then $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = P + Q$

Now $P^T = \frac{1}{2}(A + A^T)^T$

$= \frac{1}{2}(A^T + A) = P$ [using $(A + B)^T = A^T + B^T\ and\ (A^T)^T = A$]

$\therefore$ P is symmetric matrix.

$Q^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}(A^T - A) = -\frac{1}{2}(A - A^T) = -Q$

$\therefore$ Q is skew $-$ symmetric matrix.

Thus every square matrix A can be expressed as the sum of symmetric matrix $\frac{1}{2}(A + A^T)$ and skew-symmetric matrix $\frac{1}{2}(A - A^T)$.

If possible let,

$A = R + S$ ,where R is symmetric and S is skew-symmetric.

Now $A^T = (R + S)^T = R^T + S^T = R - S$

$$\therefore \mathrm{A} + A^T = 2R \Rightarrow R = \frac{1}{2}(A + A^T) = P$$

$$\mathrm{A} - A^T = 2S \Rightarrow S = \frac{1}{2}(A - A^T) = Q$$

Hence the expression is unique.

Example: Express the matrix $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

Sol$^n$: We know for a square matrix A symmetric matrix is $P = \frac{1}{2}(A + A')$ and skew-symmetric matrix is $Q = \frac{1}{2}(A - A')$

Now $P = \frac{1}{2}(A + A') = \frac{1}{2}\left\{\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}\right\}$

$$= \frac{1}{2}\begin{pmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{pmatrix} = \begin{pmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{pmatrix}$$

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left\{\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{pmatrix}$$

Now $P + Q = \begin{pmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = A$

Example: If $A = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and $I$ is the identity matrix of order 2,show that $I + A = (I - A)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$.

Proof: LHS= $I+A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{pmatrix}=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}$

RHS=$(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}=\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & -\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\\ \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\end{pmatrix}$

$$=\frac{1}{1+\tan^2\frac{\alpha}{2}}\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}1-\tan^2\frac{\alpha}{2} & -2\tan\frac{\alpha}{2}\\ 2\tan\frac{\alpha}{2} & 1-\tan^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\cos^2\frac{\alpha}{2}\begin{pmatrix}1-\tan^2\frac{\alpha}{2}+2\tan^2\frac{\alpha}{2} & -2\tan\frac{\alpha}{2}+\tan\frac{\alpha}{2}-\tan^3\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2}+\tan^3\frac{\alpha}{2}+2\tan\frac{\alpha}{2} & 2\tan^2\frac{\alpha}{2}+1-\tan^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\cos^2\frac{\alpha}{2}\begin{pmatrix}1+\tan^2\frac{\alpha}{2} & -\tan\frac{\alpha}{2}(1+\tan^2\frac{\alpha}{2})\\ \tan\frac{\alpha}{2}(1+\tan^2\frac{\alpha}{2}) & 1+\tan^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\cos^2\frac{\alpha}{2}\begin{pmatrix}\sec^2\frac{\alpha}{2} & -\tan\frac{\alpha}{2}\sec^2\frac{\alpha}{2}\\ \tan\frac{\alpha}{2}\sec^2\frac{\alpha}{2} & \sec^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\cos^2\frac{\alpha}{2}\sec^2\frac{\alpha}{2}\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}$$

$\therefore LHS=RHS$

**Adjoint of a square matrix:** Adjoint of a square matrix A is the transpose of the matrix obtained by replacing all its elements by their corresponding co-factors and is denoted by adjA.

**Steps to find the adjoint of a square matrix A:**

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

Step1:Find the cofactors $C_{11}, C_{12}, C_{13}$ etc. of the elements $a_{11}, a_{12}, a_{13}$ etc.of the determinant $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Step2:Make the matrix with corresponding co-factors i.e

$\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$,which is called co-factor matrix of matrix A.

Step3: take the transpose of co-factor matrix obtained in step2 and the matrix thus obtained is called adjoint of matrix A.

$\therefore adjA = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

**Properties of adjoint matrix:**

(i) A$(adjA) = (adjA)A = |A|I_n$ ,where $n$ is the order of matrix.

(called Jacobi's theorem)

(ii) $|adjA| = |A|^{n-1}$ , where $n$ is the order of matrix.

(iii) A$(adjA) = (adjA)A = 0$ ,if $|A| = 0$

(iv) $adj(AB) = (adjB)\,(adjA)$

(v) $(adjA^n) = (adjA)^n$, where $n \in N$

(vi) $dj(\lambda A) = \lambda^{n-1} adjA$ , where $n$ is the order of matrix.

(vii) $adj(A^T) = (adjA)^T$

(viii) $adj(adjA) = |A|^{n-2}A$, where $n$ is the order of matrix.

(ix) $|adj(adjA)| = |A|^{(n-1)^2}$, where $n$ is the order of matrix.

Proof(i): $A(adjA) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

$$= \begin{pmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{pmatrix}$$

$[Since\ a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = |A|$ and $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0]$

$$= |A|\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A|I$$

Similarly $(adjA)A = |A|I$

$\therefore A(adjA) = (adjA)A = |A|I_n$

Proof(ii): We know $A(adjA) = |A|I_n = \begin{pmatrix} |A| & 0 & 0 \cdots & 0 \\ 0 & |A| & \ldots & 0 \\ 0 & \ldots & & 0 \\ 0 & \ldots & & \\ 0 & \ldots & & |A| \end{pmatrix} = |A|^n I_n$

Now $|A(adjA)| = ||A|^n I_n|$

$\Rightarrow |A||adjA| = |A|^n|I_n| = |A|^n$

$\Rightarrow |adjA| = \frac{|A|^n}{|A|} = |A|^{n-1}\ ,if\ |A| \neq 0$

(vi) $adj(\lambda A) = \lambda^{n-1}adjA$

Verification: Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

Then $adjA = \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} \\ \lambda a_{21} & \lambda a_{22} \end{pmatrix}$$

$$adj(\lambda A) = \begin{pmatrix} \lambda a_{22} & -\lambda a_{12} \\ -\lambda a_{21} & \lambda a_{11} \end{pmatrix} = \lambda \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = \lambda^{2-1} adjA$$

Example: Find $adjA$ for $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Sol$^n$: Co-factor of $2(a_{11}) = 4$

Co-factor of $3(a_{12}) = -1$

Co-factor of $1(a_{21}) = -3$

Co-factor of $4(a_{22}) = 2$

$\therefore\ adjA = \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$

**Note**: For any square matrix $A$ of order 2 ,$adjA$ can be written directly ,simply by interchanging the elements of principal diagonal and changing the sign of elements belonging to secondary diagonal.

Example: Find the adjoint of matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$

Sol$^n$: $cofactor\ of\ 0(a_{11}) = (8 - 0) = 8$

$cofactor\ of\ 1(a_{12}) = -(4 - 0) = -4$

$cofactor\ of\ 1(a_{13}) = (-1 - 6) = -7$

$cofactor\ of\ 1(a_{21}) = -(4 + 1) = -5$

$cofactor\ of\ 2(a_{22}) = (0 - 3) = -3$

$cofactor\ of\ 0(a_{23}) = -(0 - 3) = 3$

$cofactor\ of\ 3(a_{31}) = (0 - 2) = -2$

$cofactor\ of - 1(a_{32}) = -(0 - 1) = 1$

$cofactor\ of\ 4(a_{33}) = (0 - 1) = -1$

$$\therefore\ adjA = \begin{pmatrix} 8 & -5 & -2 \\ -4 & -3 & 1 \\ -7 & 3 & -1 \end{pmatrix}$$

Example: $A$ is a square matrix of order 3 and $|A| = 7$,write the value of $|adjA|$.

Sol$^n$: $|adjA| = |A|^{3-1} = |A|^2 = 7^2 = 49.$

Example: If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$,then A$(adjA)$ is **(2009-I)**

(a)$\begin{pmatrix} 0 & 10 \\ 10 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 10 \\ 10 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 10 & 1 \\ 1 & 10 \end{pmatrix}$

Sol$^n$: A$(adjA) = |A|I = 10\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$

Ans (b)

Example: The adjoint of matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$ is **(2017-II)**

(a) $\begin{pmatrix} 1 & 6 & 2 \\ -2 & 1 & -4 \\ 6 & 3 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 6 & 1 & 2 \\ 4 & -1 & 2 \\ 6 & 3 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -6 & 2 & 1 \\ 4 & -2 & 1 \\ 3 & 1 & -6 \end{pmatrix}$

Sol$^n$: $|A| = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix} = 1 + 2(6-0) = 13$

$$|A|I = \begin{pmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{pmatrix}$$

Using hit and trial method $\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1 \end{pmatrix} = |A|I$

∴Ans (b)

**Inverse of a square matrix:** A square matrix A of order $n$ is said to be invertible ,if there exists another square matrix B of same order such that

$$AB = BA = I_n$$

Then B is called inverse of A and is denoted by $A^{-1}$.

Again we know ,

$A(adjA) = (adjA)A = |A|I_n$

$\Rightarrow \frac{A(adjA)}{|A|} = \frac{(adjA)A}{|A|} = I_n$ [dividing by $|A| if\ |A| \neq 0$]

$$\Rightarrow A \times \frac{(adjA)}{|A|} = \frac{(adjA)}{|A|} \times A = I_n$$

Hence by definition of inverse of a matrix $A^{-1} = \frac{(adjA)}{|A|}$ ,if $|A| \neq 0$

**Inverse of a square matrix (if exist) is unique:**

Proof: If possible let B and C be two inverse of A.

Then by definition of inverse of a matrix

$AB = BA = I$ and

$AC = CA = I$

Now $B = BI = B(AC) = (BA)C = IC = C$

Thus inverse of square matrix is unique.

Example: If $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$,find $A^{-1}$.

Sol$^n$: We know $A^{-1} = \frac{adjA}{|A|}$ , $|A| \neq 0$

Now $|A| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3-0) - 2(-1-0) - 2(2-0) = 1$

$\therefore\ A^{-1}$exist

$cofactor\ of\ 1(a_{11}) = (3-0) = 3$

$cofactor\ of\ 2(a_{12}) = -(-1-0) = 1$

$cofactor\ of - 2(a_{13}) = (2-0) = 2$

$cofactor\ of - 1(a_{21}) = -(2-4) = 2$

$cofactor\ of\ 3(a_{22}) = (1-0) = 1$

$cofactor\ of\ 0(a_{23}) = -(-2-0) = 2$

$cofactor\ of\ 0(a_{31}) = (0+6) = 6$

$cofactor\ of -2(a_{32}) = -(0-2) = 2$

$cofactor\ of\ 1(a_{33}) = (3+2) = 5$

$$\therefore\ adjA = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$

$$\therefore\ A^{-1} = \frac{adjA}{|A|} = \frac{1}{1}\begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$

**Properties of inverse matrix:**

(i) $AA^{-1} = I$

(ii) $(A^{-1})^{-1} = A$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A^n)^{-1} = (A^{-1})^n$

(v) $(A^{-1})^T = (A^T)^{-1}$

(vi) $|A^{-1}| = \frac{1}{|A|}$

(vii) $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$, where $\lambda \in R$

(viii) If A is symmetric matrix then $A^{-1}$ is also symmetric .

Proof(ii) : We know

$AA^{-1} = I$

$\therefore$ By definition of inverse of a matrix $(A^{-1})^{-1} = A$

Proof(iii) : $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$

Again $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = B^{-1}B = I$

$\therefore (AB)(B^{-1}A^{-1}) = (B^{-1}A^{-1})(AB) = I$

Hence by definition of inverse of a matrix $(\mathrm{AB})^{-1} = \mathrm{B}^{-1}\mathrm{A}^{-1}$

Proof(vi) : We know $AA^{-1} = I$

$\Rightarrow |AA^{-1}| = |I|$

$\Rightarrow |A||A^{-1}| = 1$

$\Rightarrow |A^{-1}| = \dfrac{1}{|A|}$

Proof(vii) : $(\lambda A)^{-1} = \frac{adj(\lambda A)}{|\lambda A|} = \frac{\lambda^{n-1} adjA}{\lambda^n |A|}$ [using properties of adjoint]

$$= \frac{1}{\lambda} \times \frac{adjA}{|A|} = \lambda^{-1} A^{-1}$$

*Example*: For a square matrix A ,which of the following properties hold?

1. $(A^{-1})^{-1} = A$ (2) $det(A^{-1}) = \frac{1}{detA}$ (3) $(\lambda A)^{-1} = \lambda A^{-1}$, where $\lambda \in R$

Select the correct answer using the code given below.**(2018-II)**

(a)1 and 2 only (b) 2 and 3 only (c) 1 and 3 only (d) 1,2 and 3

Ans (a)

*Example*: If $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ $and$ $B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$,then what is $B^{-1}A^{-1}$?

(a) $\begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 3 \\ -1 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & -3 \\ 1 & -2 \end{pmatrix}$

Sol$^n$: Here $AB = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$

$B^{-1}A^{-1} = (AB)^{-1} = \frac{adj(AB)}{|AB|} = \frac{\begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}}{-1} = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$

Ans (b)

**Solution of system of linear equations by matrix method:**

Let us consider a system of linear equation in three variables $x, y, z$ as

$$\left.\begin{matrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{matrix}\right\} \rightarrow (1)$$

The matrix equation of the system (1) is

$AX = B \rightarrow (2)$

Where $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}, B = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

If $|A| \neq 0$, then $A^{-1}$exists and from (2) we get

$A^{-1}(AX) = A^{-1}B$

$\Rightarrow (A^{-1}A)X = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^{-1}B$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

$\Rightarrow \left.\begin{matrix} x = \alpha \\ y = \beta \\ z = \gamma \end{matrix}\right\}$ which is the required solutions.

**Solution of matrix equation $\mathbf{AX = B}$ is unique:**

Proof: If possible let $X_1$ and $X_2$ be two solutions of $\mathrm{AX = B}$.

Then $\mathrm{AX_1 = B}$ and $\mathrm{AX_2 = B}$

$\therefore \ \mathrm{AX_1 = AX_2}$

$\Rightarrow A^{-1}(\mathrm{AX_1}) = A^{-1}(\mathrm{AX_2})$

$\Rightarrow (A^{-1}A)X_1 = (A^{-1}A)X_2$

$\Rightarrow IX_1 = IX_2$

$\Rightarrow X_1 = X_2$

$\therefore$ Solution of matrix equation is unique.

Example: Solve the system of equations by matrix method.

$x + y + z = 6$

$x + 2z = 7$

$3x + y + z = 12$

Sol$^n$: The matrix equation of the system is

$X = A^{-1}B \rightarrow (1)$

Where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 6 \\ 7 \\ 12 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix} = -1(1-6) - 1(2-1) = 4 \neq 0$$

$\therefore\ A^{-1}$exists.

$cofactor\ of\ 1(a_{11}) = (0-2) = -2$

$cofactor\ of\ 1(a_{12}) = -(1-6) = 5$

$cofactor\ of 1(a_{13}) = (1-0) = 1$

$cofactor\ of 1(a_{21}) = -(1-1) = 0$

$cofactor\ of\ 0(a_{22}) = (1-3) = -2$

$cofactor\ of\ 2(a_{23}) = -(1-3) = 2$

$cofactor\ of\ 3(a_{31}) = (2-0) = 2$

$cofactor\ of 1(a_{32}) = -(2-1) = -1$

$cofactor\ of\ 1(a_{33}) = (0-1) = -1$

$\therefore\ adjA = \begin{pmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{pmatrix}$

$\therefore\ A^{-1} = \frac{adjA}{|A|} = \frac{1}{4}\begin{pmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{pmatrix}$

Now from (1) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{pmatrix}\begin{pmatrix} 6 \\ 7 \\ 12 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 12 \\ 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$\therefore x = 3, y = 1, z = 2$

Example: Use the product $\begin{pmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{pmatrix}\begin{pmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{pmatrix}$ to solve the system of equations

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$

Sol$^n$: Let $P = \begin{pmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{pmatrix}$ $and\ Q = \begin{pmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{pmatrix}$

$$PQ = \begin{pmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{pmatrix}\begin{pmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{pmatrix}$$

$$= \begin{pmatrix}-2-9+12 & 0-2+2 & 1+3-4\\0+18-18 & 0+4-3 & 0-6+6\\-6-18+24 & 0-4+4 & 3+6-8\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} = I$$

$\therefore P^{-1} = Q$

The matrix equation of the system is

$X = A^{-1}B \rightarrow (1)$

Where $A = \begin{pmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{pmatrix} = P, B = \begin{pmatrix}1\\1\\2\end{pmatrix}, X = \begin{pmatrix}x\\y\\z\end{pmatrix}$

$\Rightarrow A^{-1} = P^{-1} = Q$

$\therefore$ From (1) $X = QB = \begin{pmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{pmatrix}\begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}0\\5\\3\end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}$$

$\therefore x = 0, y = 5, z = 3$

Example: For matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$,show that $A^2 - 5A + 4I = 0$ and hence find $A^{-1}$.

Sol$^n$: $A^2 = A \times A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$$

$$\therefore A^2 - 5A + 4I = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

Now $A^2 - 5A + 4I = 0$

$\Rightarrow A^{-1}(A^2 - 5A + 4I) = A^{-1}0$

$\Rightarrow (A^{-1}A)A - 5(A^{-1}A) + 4A^{-1}I = 0$

$\Rightarrow IA - 5I + 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = 5I - A$

$$\Rightarrow 4A^{-1} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

Example: If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$,show that $F(x)F(y) = F(x+y)$,

Hence show that $[F(x)]^{-1} = F(-x)$

Proof: $F(x)F(y) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{pmatrix} = F(x+y)$$

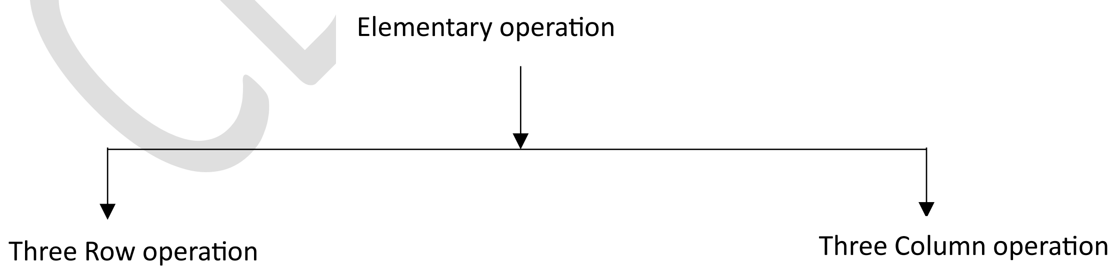
Now $F(x)F(y) = F(x+y)$

$\Rightarrow \; F(x)F(-x) = F(x-x) = F(0) \quad [putting\ y = -x\ both\ sides]$

$$\Rightarrow F(x)F(-x) = \begin{pmatrix} \cos 0 & -\sin 0 & 0 \\ \sin 0 & \cos 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$\therefore [F(x)]^{-1} = \; F(-x)$

**Elementary transformation(or operation)of a matrix**:




**Row operation:**

1 .interchange of any two rows ,denoted by $R_1 \leftrightarrow R_2$

2.multiplication of elements of any row by a number $\lambda(\neq 0)$,denoted by

$R_1 \to \lambda R_1$

3. addition of elements of any row with the corresponding elements of any other row multiplied by a number $\lambda(\neq 0)$,denoted by $R_1 \to R_1 + \lambda R_2$

**Column operation:**

1 .interchange of any two columns ,denoted by $C_1 \leftrightarrow C_2$

2.multiplication of elements of any column by a number $\lambda(\neq 0)$,denoted by

$C_1 \to \lambda C_1$

3. addition of elements of any column with the corresponding elements of any other column multiplied by a number $\lambda(\neq 0)$,denoted by $C_1 \to C_1 + \lambda C_2$

**Note:** (i) matrix obtained by using either row operation or column operation is called equivalent matrix of the given matrix.

(ii) both operations can't be mixed up till the end of the sum.

**Inverse of a matrix using elementary operation:**

**For row operation :** write $A = IA$ and use elementary row operations on $A$ of LHS and on $I$ of RHS, till we get $I = BA$, then $A^{-1} = B$.

**For row operation :** write $A = AI$ and use elementary row operations on $A$ of LHS and on $I$ of RHS, till we get $I = AB$, then $A^{-1} = B$.

Example: Find $A^{-1}$ using elementary operation, where $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

Sol$^n$: we know $A = IA$

$$\Rightarrow \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \qquad \text{using } R_1 \leftrightarrow R_2$$

$$\Rightarrow \begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 0 & -5 & -8\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & -3 & 1\end{pmatrix}A \qquad \text{using } R_3 \to R_3 - 3R_1$$

$$\Rightarrow \begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 2\\ 0 & 0 & 2\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 5 & -3 & 1\end{pmatrix}A \qquad \text{using } R_3 \to R_3 + 5R_2$$

$$\Rightarrow \begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & 0 & 2\end{pmatrix} = \begin{pmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ 5 & -3 & 1\end{pmatrix}A \qquad \text{using } R_1 \to R_1 - 2R_2$$

$$\Rightarrow \begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}-2 & 1 & 0\\ 1 & 0 & 0\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{pmatrix}A \qquad \text{using } R_3 \to \frac{1}{2}R_3$$

$$\Rightarrow \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 2\\ 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & \frac{-1}{2} & \frac{1}{2}\\ 1 & 0 & 0\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{pmatrix}A \qquad \text{using } R_1 \to R_1 + R_3$$

$$\Rightarrow \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & \frac{-1}{2} & \frac{1}{2}\\ -4 & 3 & -1\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{pmatrix}A \qquad \text{using } R_2 \to R_2 - 2R_3$$

$I = BA$

$$\therefore A^{-1} = B = \begin{pmatrix}\frac{1}{2} & \frac{-1}{2} & \frac{1}{2}\\ -4 & 3 & -1\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{pmatrix}$$

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## Practice Questions

### Section-I (1 mark each)

**MCQ type**

1. If A & B are symmetric matrices of same order,then ( AB – BA)is

(a) Skew − symmetric (b) symmetric − matrix
(c) Zero − matrix (d) Identity matrix

Ans (a)

2. *If* A= $\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ is such that $A^2 = I$ then

$1+\alpha^2 + \beta\gamma = 0$ (b) $1-\alpha^2 + \beta\gamma = 0$ ( c) $1 -\alpha^2 - \beta\gamma = 0$ (d)$1+\alpha^2 - \beta\gamma = 0$

Ans ( c)

3. If the matrix A is both symmetric and skew -symmetric ,then

(a) A is a diagonal matrix (b) A is a zero matrix

( c) A is a square matrix (d) None of these

Ans (b)

4. If A is a square matrix such that $A^2 = A$ ,then $(I + A)^3 - 7A$ is equal to

(a) A (b) I − A ( c) I (d) 3A

Ans ( c)

5. If a,b,c are in AP , then the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

(a) 0 (b ) 1 ( c) x (d) 2x

Ans (a)

6. If x,y,z are nonzero real numbers ,then the inverse of matr*ix A* =$\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ *is*

(a) $\begin{pmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{pmatrix}$ (b) xyz$\begin{pmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{pmatrix}$ ( c)$\frac{1}{xyz}\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ (d)$\frac{1}{xyz}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Ans (a)

7. If $A = [a_{ij}]$ is a square matrix of even order such that $a_{ij} = i^2 - j^2$,then

(a) A is a skew-symmetric matrix and $|A| = 0$

(b) A is a symmetric matrix and $|A|$ *is a* square.

(c) A is a symmetric matrix and $|A| = 0$

(d) none of these.

8. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$ then $x + y$ equals to

$(a)\ 0$ $(b)\ -1$ (c) 2 (d) none of these

9.Let A = $\begin{pmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{pmatrix}$ where $0 \le \theta \le 2\pi$ ,then

(a) $|A| = 0$ (b) $|A| \in (2, \infty)$ ( c) $|A| \in (2,4)$ (d) $|A| \in [2,4]$

Ans (d)

## Assertion –Reason Based Questions

In the following questions ,a statement of Assertion (A) is followed by a statement of Reason (R) Choose the correct answer out of the following choices

(a) Both A and R true and R is the correct explanation of A .

(b) Both A and R true and R is not the correct explanation of A .

(c ) A is true but R is false .

(d)A is false but R is true .

1.Assertion(A) : If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then $x = \pm 6$

Reason(R): If A and B are matrices of order 3 and $|A| = 4, |B| = 6\ then\ |2AB| = 192$

Ans: (b)

2. Assertion(A) : If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}, then\ x = 2, y = 2, z = -5\ and\ w = 4$

Reason(R): Two matrices are equal ,if their orders are same and their corresponding elements are equal.

Ans: (a)

3. Assertion(A) : If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}, then\ |A| = 0$

Reason(R): $|adjA| = |A|^{n-1}$,where $n$ is the order of matrix.

Ans: (b)

4. Assertion(A) : If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$,then AB and BA both are defined .

Reason(R): For two matrices A and B ,the product AB is defined ,if the number of columns in A is equal to the number of rows in B.

Ans: (a)

5. Let A and B be two symmetric matrices of order 3

Assertion(A) :A(BA) and (AB)A are symmetric matrices.

Reason(R): AB is symmetric matrix ,if multiplication of A with B is commutative.

Ans: (b)

## Section-II(2 mark each)

1. Prove that diagonal elements of a skew –symmetric matrix are all zeros.

2.If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ $and\ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ find k so that $A^2 = 5A + KI$

Find a matrix A such that 2A – 3B + 5C = O, where B = $\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

## Section-III(5 mark each)

1. Find matrix $X$ so that $X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

2 .If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$,find $A^{-1}$. Using $A^{-1}$ solve the systemof equation

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$$

3. Use product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ to solve the system of equations: $x + 3z = 9, -x + 2y - 2z = 4, 2x - 3y + 4z = -3.$

4. Use product $\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$ to solve the system of equations: $x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$

5. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$ $find\ A^{-1}$

   Hence, solve the system of equations:

   $3x + 3y + 2z = 1,\ \ x + 2y = 4,\ 2x - 3y - z = 5$

6. Using matrices solve the following system of equations:-

$$\begin{aligned} 3x + \quad 4y + \quad 7z &= \ \ 4 \\ 2x - \quad y + \quad 3z &= -3 \\ x + \quad 2y - \quad 3z &= \ \ 8 \end{aligned}$$

7. If A=$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$,find $A^{-1}$.Hence solve the following system of equations:

$x + 2y - 3z = -4, 2x + 3y + 2z = 2 \ and \ 3x - 3y - 4z = 11$

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## CHAPTER-4 DETERMINANTS

### Application of Determinants:

(i) **Used to find the area of a triangle if vertices are given.**

(ii) **Used to check the consistency of system of linear equations of two or more variables and to find their solutions.**

(iii) **Used in vector algebra and three dimensional co-ordinate geometry.**

(iv) **Used in different branches of engineering, social science, economics etc.**

**Determinant of order 2 (or second order determinant)**: It is a number (real or complex), which is expressed in terms of elements (real or complex) arranged in the form of two rows and two columns and enclosed them within a pair of vertical lines.

$$\begin{array}{c} \quad C_1 \quad\; C_2 \\ \quad\downarrow \quad\;\; \downarrow \\ \begin{vmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{vmatrix} \begin{array}{l} \longrightarrow R_1 \\ \longrightarrow R_2 \end{array} \end{array}$$

Note : (i) The suffixes $i, j$ of the elements $a_{ij}$ denote the row and column in which the element $a_{ij}$ belongs.

(ii) Element $a_{11}$ belongs to first row and first column and is called the leading element of the determinant.

(iii) To find the value of a second order determinant ,use the rule

$$(multiply\ leading\ element\ with\ the\ element\ belong\ to\ leading\ diagonal) - (multiply\ the\ elements\ belonging\ to\ secondary\ diagonal)$$

Example 1: Find the value of the determinant $\begin{vmatrix} 2 & 3 \\ 1 & -5 \end{vmatrix}$

Sol$^n$: $\begin{vmatrix} 2 & 3 \\ 1 & -5 \end{vmatrix} = 2(-5) - 3 \times 1 = -13$

Example2: Find the value of the determinant $\begin{vmatrix} 2+i & 3i \\ 2i-1 & 4 \end{vmatrix}$

Sol$^n$: $\begin{vmatrix} 2+i & 3i \\ 2i-1 & 4 \end{vmatrix} = (2+i)4 - 3i(2i-1)$

$$= 8 + 4i - 6i^2 + 3i$$

$= 8 + 7i + 6$

$= 14 + 7i$

Example2: If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ ,find $x$.

Sol$^n$: $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

**Determinant of order 3 (or third order determinant)**: It is a number (real or complex), which is expressed in terms of elements (real or complex) arranged in the form of three rows and three columns and enclosed them within a pair of vertical lines.

$$\begin{matrix} C_1 & C_2 & C_3 & \\ \downarrow & \downarrow & \downarrow & \\ \end{matrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix}$$

**Minor and co-factor of the elements of a determinant:**

**Minor of an element :** It is the value of the determinant obtained by eliminating the row and column in which the element belong.

**Co-factor of an element :** Minor of an element when multiplied with the proper sign(i.e with +ve sign if sum of suffixes is even or with -ve sign if sum of suffixes is odd) is called co-factor of the element.

Let us consider a determinant $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 5 & 6 \\ 0 & 4 & 1 \end{vmatrix}$

| element | corresponding minor | corresponding co-factor |
|---|---|---|
| $1(= a_{11})$ | $\begin{vmatrix} 5 & 6 \\ 4 & 1 \end{vmatrix} = -19(= M_{11})$ | $-19(= C_{11})$ |
| $2(= a_{12})$ | $\begin{vmatrix} -2 & 6 \\ 0 & 1 \end{vmatrix} = -2(= M_{12})$ | $2(= C_{12})$ |
| $3(= a_{13})$ | $\begin{vmatrix} -2 & 5 \\ 0 & 4 \end{vmatrix} = -8(= M_{13})$ | $-8(= C_{13})$ |
| $-2(= a_{21})$ | $\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = -10(= M_{21})$ | $10(= C_{21})$ |

| | | |
|---|---|---|
| $5(=a_{22})$ | $\begin{vmatrix}1 & 3\\0 & 4\end{vmatrix}=4(=M_{22})$ | $4(=C_{22})$ |
| $6(=a_{23})$ | $\begin{vmatrix}1 & 2\\0 & 4\end{vmatrix}=4(=M_{23})$ | $-4(=C_{23})$ |
| $0(=a_{31})$ | $\begin{vmatrix}2 & 3\\5 & 6\end{vmatrix}=-3(=M_{31})$ | $-3(=C_{31})$ |
| $4(=a_{32})$ | $\begin{vmatrix}1 & 3\\-2 & 6\end{vmatrix}=11(=M_{32})$ | $-11(=C_{32})$ |
| $1(=a_{33})$ | $\begin{vmatrix}1 & 2\\-2 & 5\end{vmatrix}=9(=M_{33})$ | $9(=C_{33})$ |

**Working rules to find the value of a third order determinant:**

**Step 1:** Select any row or column only (but not a diagonal or any ) and multiply each elements of the selected row or column by their corresponding co-factor.

**Step 2:** Take the sum of results obtained in Step1,which is the value of the determinant.

**Note:**(i) Prefer to expand by a row or a column having maximum number of zeros.

(ii) If we multiply each elements of any row (or column) by its corresponding co-factors and add ,we get the value of the determinant.

i.e $a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}=value\ of\ determinant$

(iii) If we multiply each elements of any row (or column) by co-factors of any other row(or column),then their sum is zero.

Proof: Let $\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{vmatrix}$

Now $a_{11}C_{21}+a_{12}C_{22}+a_{13}C_{23}$

$$=a_{11}(-)\begin{vmatrix}a_{12} & a_{13}\\a_{32} & a_{33}\end{vmatrix}+a_{12}\begin{vmatrix}a_{11} & a_{13}\\a_{31} & a_{33}\end{vmatrix}+a_{13}(-)\begin{vmatrix}a_{11} & a_{12}\\a_{31} & a_{32}\end{vmatrix}$$

$$=-a_{11}(a_{12}a_{33}-a_{13}a_{32})+a_{12}(a_{11}a_{33}-a_{13}a_{31})-a_{13}(a_{11}a_{32}-a_{12}a_{31})$$

$$=-a_{11}a_{12}a_{33}+a_{11}a_{13}a_{32}+a_{12}a_{11}a_{33}-a_{12}a_{13}a_{31}-a_{13}a_{11}a_{32}+a_{13}a_{12}a_{31}$$

$$=0$$

Example3: Evaluate the determinant $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

Sol$^n$: $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

$= 3(1+6) + (-4)(-)(1+4) + 5(3-2)$ (expanding by first row)

$= 21 + 20 + 5 = 46$

Example4: Evaluate the determinant $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

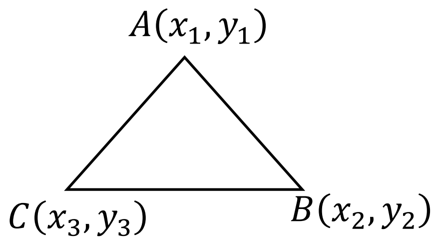
Sol$^n$: $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$= 0 + 0 + (-1)(-)(-15+3) = -12$ (expanding by second row)

§ **Area of a Triangle:** Let $ABC$ be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Then the area of triangle $ABC$ is the absolute value of the expression

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$




This expression can also be written in the form of a determinant

$$\mathbf{\Delta} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

**Remark:** If area of triangle is given (which is absolute value) ,for finding other information, both positive and negative values of area to be use.

Example5: Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants and find $k$ if $D(k, 0)$ is a point such that area of triangle $ABD$ is 3sq units.

Sol$^n$: Let $P(x, y)$ be any point on the line AB .

Then $\Delta \mathrm{APB} = 0$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow -1(y - 3x) = 0$$

$$\Rightarrow y = 3x$$

Which is the required equation of line AB.

Again $\Delta \mathrm{ABD} = 3$

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3$$

$$\Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 6$$

$$\Rightarrow -(-3k) = \pm 6$$

$$\Rightarrow k = \pm 2$$

Example6:If the area of triangle is 35 sq units with vertices $(2, -6), (5,4)$ and $(k, 4)$,then $k$ is

(a) 12 (b) $-2$ (c) $-12, -2$ (d) $12, -2$

Sol$^n$: $\Delta \mathrm{ABC} = 35$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35$$

$$\Rightarrow \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 70$$

$$\Rightarrow 2(4 - 4) + (-6)(-)(5 - k) + 1(20 - 4k) = \pm 70$$

$$\Rightarrow 30 - 6k + 20 - 4k = \pm 70$$

$\Rightarrow -10k = \pm 70 - 50$

$\Rightarrow -10k = 20, -120$

$\Rightarrow k = -2, 12$

$Ans\ (d)$

## § Properties of Determinants :

$P_1$: The value of a determinant remain same if its rows and columns are interchanged.

Verification: $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = 2(-20) + 3(-42-4) + 5(30)$

$= -40 - 138 + 150 = -28$

$\Delta_1 = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$ [interchanging rows and columns of $\Delta$]

$= 2(-20) - 6(21-25) + (-12) = -40 + 24 - 12 = -28$

$\therefore\ \Delta = \Delta_1$

$P_2$: If any two rows ( or columns) of a determinant are interchanged ,then the determinant changes its sign.

Verification: $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = 2(-20) + 3(-42-4) + 5(30)$

$= -40 - 138 + 150 = -28$

$\Delta_1 = \begin{vmatrix} 5 & -3 & 2 \\ 4 & 0 & 6 \\ -7 & 5 & 1 \end{vmatrix}$ [using $C_1 \leftrightarrow C_3$ of $\Delta$]

$= 5(-30) + 3(4+42) + 2(20) = -150 + 138 + 40 = 28$

$\therefore\ \Delta = -\Delta_1$

$P_3$: If all the elements of any row (or any column) have a common factor m ,then m is a factor of the determinant.

i.e $\Delta = \begin{vmatrix} ma_1 & mb_1 & mc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = m \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$P_4$: A determinant vanishes (i.e zero) if any two rows ( or any two columns) are same or identical.

Verification: $\Delta = \begin{vmatrix} 2 & -4 & 3 \\ 1 & 3 & 4 \\ 2 & -4 & 3 \end{vmatrix} = 2(9+16) + 4(3-8) + 3(-4-6)$

$$= 50 - 20 - 30 = 0 \ [\because C_1 = C_3]$$

$P_5$: If each elements (or some element) of a row (or a column ) is expressed as the sum of two or more quantities ,then the determinant can be expressed as the sum of two or more determinants .

i.e $\begin{vmatrix} a_1+\alpha_1 & b_1 & c_1 \\ a_2+\alpha_2 & b_2 & c_2 \\ a_3+\alpha_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$

$P_6$: The value of a determinant remain same ,if any row (or column) is multiplied by a number and is then added to (or subtracted from) any other row (or column).

Verification: $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_1 = \begin{vmatrix} a_1+\lambda b_1 & b_1 & c_1 \\ a_2+\lambda b_2 & b_2 & c_2 \\ a_3+\lambda b_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda b_1 & b_1 & c_1 \\ \lambda b_1 & b_2 & c_2 \\ \lambda b_1 & b_3 & c_3 \end{vmatrix}$ [using $C_1=C_1+\lambda C_2$]

$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \lambda \begin{vmatrix} b_1 & b_1 & c_1 \\ b_1 & b_2 & c_2 \\ b_1 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + 0$ [using $P_3$ and $P_4$]

$= \Delta$

$P_7$: If $D$ is the value of a determinant of order $n$ and $D'$is the value of determinant of the corresponding co-factors ,then $DD' = D^n$

**Note**: (i) At least one row or one column should remain unchanged.

(ii) $C_i = C_i + mC_j + nC_k \ where\ j,k \neq i$

(iii) $R_i = R_i + mR_j + nR_k \ where\ j,k \neq i$

§ **Working rules:**

1.Common factor in a row(s) or column (s) to be taken outside the determinant.

2. If possible make two of the elements in a row(or column) as zero.

**Cramer's Rule:** It is used to find the solution of a system of linear equation in two or more variables.

Let us consider a system of linear equation in three variables $x, y, z$.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Then by Cramer's rule

$$\left.\begin{aligned} x &= \frac{D_x}{D} \\ y &= \frac{D_y}{D} \\ z &= \frac{D_z}{D} \end{aligned}\right\}$$

Where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, $D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

**Case1:** If $D \neq 0$ and at least one of $D_x, D_y, D_z$ is non -zero ,the system is consistent and it has a unique solution.

**Case2:** If $D \neq 0$ ,but $D_x = D_y = D_z = 0$,the system is consistent and it has a trivial solution (i.e $x = y = z = 0$)

**Case3:** If $D = D_x = D_y = D_z = 0$,then the system is consistent and it has infinite number of solutions.

**Case4:** If $D = 0$ but at least one of $D_x, D_y, D_z$ is non-zero, the system is

in-consistent and it has no solution.

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## Practice Questions

### Section-I(1 mark each)

1. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$ *then write the value of* $x$.
2. Evaluate : $\begin{vmatrix} \sin 30° & \cos 30° \\ -\sin 60° & \cos 60° \end{vmatrix}$
3. Evaluate $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$.
4. Write the value of following determinant: $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

5. What positive value of x makes the following pair of determinants equal?

$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

6. Evaluate the determinant Δ = $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$

7 .If there are two values of $'a'$ which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$ , then find the sum of these values.

## Section-II(4mark each)

1. Using properties of determinants, prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

2. Using properties of determinant, prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc+bc+ca+ab$$

3. Prove that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

4. Using properties of determinant ,prove that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2.$$

5. Using properties of determinant prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

6. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$$

7. If x,y,z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, \text{ then show that } 1+xyz=0$$

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<u>**Sainik School Imphal**</u>

<u>**SUMMER VACATION WORK for NDA**</u>

1. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals (**AIEEE-2005**)
   (a)1 (b) 2 (c) 3 (d) $-2$

2. If $\alpha, \beta$ are the roots of $ax^2+bx+c=0$ and $\alpha+h, \beta+h$ are the roots of $px^2+qx+r=0$, then what is $h$ equal to? (**2014-II**)
   (a) $\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$ (b) $\frac{1}{2}\ -\frac{b}{a}+\frac{q}{p}$ (c) $\frac{1}{2}\ \frac{b}{p}+\frac{q}{a}$ (d) $\frac{1}{2}\ -\frac{b}{p}+\frac{q}{a}$

3. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$, is equal to the sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in
   (a) $A.P.$ (b) $G.P.$ (c) $H.P.$ (d) none

4. If $9^x - 4 \times 3^{x+2} + 3^5 = 0$, then the solution set is
   (a) {1,2} (b) {2,3} (c) {2,4} (d) {1,3}

5. If the equation $ax^2+bx+c=0$ & $x^2+x+1=0$ have a common root, then
   (a) $a+b+c=0$ (b) $a=b=c$ (c) $a-b+c=0$ (d) none

6. The sum of the real roots of the equation $x^2+|x|-6=0$ is
   (a) 4 (b) 0 (c) -1 (d) None

7. If the roots of the equation $ax^2+bx+c=0$ are in the ratio $1:n$, then
   (a) $na^2 = bc(1+n)^2$ (b) $nb^2 = ca(1+n)^2$
   (c) $nb^2 = ca(1-n)^2$ (d) None

8. If $\alpha, \beta$ are the roots of the quadratic equation $x^2 - 2cos\theta x + 1 = 0$, then equation whose roots are $\alpha^n$ & $\beta^n$ is
   (a) $x^2 - (2Cosn\theta)x + 1 = 0$ (b) $2x^2 - (2Cosn\theta)x - 1 = 0$
   (c) $x^2 + (2Cosn\theta)x + 1 = 0$ (d) $x^2 + (2Cosn\theta)x - 1 = 0$

9. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ & $\frac{c}{b}$ are in (**AIEEE-2003**)
   (a) Arithmetic Progression (b) Geometric Progression
   (c) Harmonic Progression (d) None

10. The least integer K which makes the roots of the equation $x^2+5x+k=0$ imaginary is
    (a) 4 (b) 5 (c) 6 (d) 7

11. If a & b are roots of $x^2 - px + q = 0$, then $\frac{1}{a}+\frac{1}{b}$ is
    (a) $\frac{1}{p}$ (b) $\frac{1}{q}$ (c) $\frac{1}{2p}$ (d) $\frac{p}{2}$

12. If $\alpha, \beta$ are the roots of the equation $x^2+x+1=0$ and $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of the equation $x^2+px+q=0$ then $p$ is
    (a) -1 (b) 1 (c) -2 (d) 2

13. If $x$ is real, then the minimum value of $x^2 - 8x + 17$ is

(a) 2 (b) 4 (c) 1 (d) 3

14. If $m$ and $n$ are the roots of the equation $(x+p)(x+q)-k=0$, then the roots of the equation $(x-m)(x-n)+k=0$ are (**2015-I**)
(a) $p$ & $q$ (b) $\frac{1}{p}$ & $\frac{1}{q}$ (c) $-p$ & $-q$ (d) $p+q$ & $p-q$

15. If $sec\ \alpha$ & $cosec\ \alpha$ are the roots of the equation $x^2 - px + q = 0$, then
(a) $p^2 = q\ (q-2)$ (b) $p^2 = q\ (q+2)$ (c) $p^2 + q^2 = 2q$ (d) $None$

16. If $2\ sin^2 \frac{\pi}{8}$ is a root of the equation $x^2 + ax + b = 0$ $where\ a$ & $b$ are rational numbers, then $a - b$ is equal to
(a) $-\frac{5}{2}$ (b) $-\frac{3}{2}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

17. The G.M of roots of the equation $x^2 - 18x + 9 = 0$ is
(a) 3 (b) 4 (c) 2 (d) 1

18. If the difference between the roots of the equation $x^2 + ax + b = 0$ is equal to the difference between the roots of the equation $x^2 + bx + a = 0$ where $a \neq b$, then
(a) $a + b = 4$ (b) $a + b = -4$ (c) $a - b = 4$ (d) $a + b = -4$

19. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then (**2015-II**)
(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = a + b$
(c) $ab + b^2 = 2ac$ (d) $ab - b^2 = 2ac$

20. If $\alpha, \beta$ are the roots of the equation $(x-a)(x-b) = c,\ c \neq 0$, then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are(**2011-II,2015-I**)
(a) $a, c$ (b) $b, c$ (c) $a, b$ (d) $a + c,\ b + c$

21. If $\alpha, \beta$ are the roots of the equation $8x^2 - 3x + 27 = 0$, then the value of

$$\left(\frac{\alpha^2}{\beta}\right)^{\frac{1}{3}} + \left(\frac{\beta^2}{\alpha}\right)^{\frac{1}{3}}$$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{3}$ (d) 4

22. If $x^2 + ax + 10 = 0$ & $x^2 + bx - 10 = 0$ have a common root, then $a^2 - b^2$ is equal to
(a) 10 (b) 20 (c) 50 (d) 40

23. If $2 + i$ is a root of the equation $x^3 - 5x^2 + 9x - 5 = 0$, then the other roots are
(a) $1$ & $2 - i$ (b) $-1$ & $3 + i$ (c) $0$ & $1$ (d) $None$

24. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtains 8 & 2 for roots. Another student makes a mistake only in the coefficient of first degree term and finds $-9$ & $-1$ for roots. The correct equation is (**2015-I**)

(a) $x^2 - 10x + 9 = 0$ (b) $x^2 + 10x + 9 = 0$
(c) $x^2 - 10x + 16 = 0$ (d) $x^2 - 8x - 9 = 0$

25. Difference between the corresponding roots of $x^2 + ax + b = 0$ & $x^2 + bx + a = 0$ is same & $a \neq b$, then **(AIEEE-2002)**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$

(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

Consider the following for the next two (02) items that follow: (**2016-II**)

$2x^2 + 3x - \alpha = 0$ has roots $-2$ and $\beta$ while the equation $x^2 - 3mx + 2m^2 = 0$ has both roots positive, where $\alpha > 0$ & $\beta > 0$

26. What is the value of $\alpha$?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

27. If $\beta$, 2, 2m are in G.P, then what is the value of $\beta\sqrt{m}$

(a) 1 (b) 2 (c) 4 (d) 6

28. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and $l$ is real, then what is the greatest value of m? (**2016-I**)

(a) $-\frac{9}{8}$ (b) $\frac{9}{8}$ (c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

29. If $\alpha$ & $\beta$ are the roots of $x^2 + qx + 1 = 0$ and $\gamma$ & $\delta$ are the roots of $x^2 + px + 1 = 0$, then the value of $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ is

(a) $p^2 - q^2$ (b) $q^2 - p^2$ (c) $p^2$ (d) $q^2$

30. The quadratic equation whose roots are sin18° and cos36° is

(a) $2x^2 - \sqrt{5}x + 1 = 0$ (b) $4x^2 - 2\sqrt{5}x + 1 = 0$

(c) $2x^2 + \sqrt{5}x + 1 = 0$ (d) $4x^2 + \sqrt{5}x + 1 = 0$

31. The roots of the equation $2a^2x^2 - 2abx + b^2 = 0$ *where* $a < 0$ *and* $b > 0$ *are* (**2014-I**)

(a) Sometimes complex (b) Always irrational

(c) Always complex (d) Always real

32. The value of m for which the equation $(1 - m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has equal roots, is

(a) 0, 3 (b) 1 (c) 2 (d) 3

33. If $x^2 - 2x + sin^2\alpha = 0$, then

(a) $x \in [-1, 1]$ (b) $x \in [0, 2]$ (c) $x \in [-2, 2]$ (d) $x \in [1, 2]$

34. If the equation $x^2 + 2x + 3\lambda = 0$ & $2x^2 + 3x + 5\lambda = 0$ have a non-zero common root, then $\lambda$ is equal to

(a) 1 (b) $-1$ (c) 3 (d) None

35. The sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares ,then

(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$ (c) $p^2 + p = 2q$ (d) None

Consider the following for the next two items that follow: (**2013-II**)

Let $\alpha$ and $\beta$ be the roots of the equation $x^2 - (1 - 2a^2)x + (1 - 2a^2) = 0$

36. Under what condition does the above equation have real roots ?

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$ (c) $a^2 \leq \frac{1}{2}$ (d) $a^2 \geq \frac{1}{2}$

37. Under what condition is $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$ ?

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$ (c) $a^2 > 1$ (d) $a^2 \in \left(\frac{1}{3}, \frac{1}{2}\right)$ $only$

38. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then which of the folling is correct? **(2021-II)**

(a) $a^2 + b^2 - 2ac = 0$ (b) $-a^2 + b^2 + 2ac = 0$

(c) $a^2 - b^2 + 2ac = 0$ (d) $a^2 + b^2 + 2ac = 0$

39. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$ . What is the sum of the absolute values of the roots ? **(2021-II)**

(a) 4 (b) 6 (c) 8 (d) 10

40. The quadratic equation $3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$ has real roots of equal magnitude and opposite sign .Which one of the following is correct ? **(2021-II)**

(a) 0< $k < \frac{5}{3}$ (b) 0< $k < \frac{3}{5}$ $only$ (c) $\frac{3}{5} < k < \frac{5}{3}$ (d) no such value of $k$ exists

41. If $p$ and $q$ are the non –zero roots of the equation $x^2 + px + q = 0$ then how many possible values can q have ? **(2021-II)**

(a) Nil (b) One (c) Two (d) Three

42. If the roots of the equation $4x^2 - (5k + 1)x + 5k = 0$ differ by unity ,then which one of the following is a possible value of $k$? **(2021-II)**

(a) $-3$ (b) $-1$ (c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

43. Let $\alpha$ $and$ $\beta$ be the roots of the equation $x^2 + px + q = 0$. If $\alpha^3$ $and$ $\beta^3$ are the roots of the equation $x^2 + mx + n = 0$ , then what is the value of $m + n$ ? **(2022-I)**

(a) $p^3 + q^3 + pq$ (b) $p^3 + q^3 - pq$ (c) $p^3 + q^3 + 3pq$ (d) $p^3 + q^3 - 3pq$

44.Let $\alpha$ *and* $\beta$ be the roots of the equation $x^2 - ax - bx + ab - c = 0$ . What is the quadratic equation whose roots are $a$ *and* $b$ ? **(2022-I)**

(a)$x^2 - \alpha x - \beta x + \alpha\beta + c = 0$ (b) $x^2 - \alpha x - \beta x + \alpha\beta - c = 0$

(c) $x^2 + \alpha x + \beta x + \alpha\beta + c = 0$ (d) $x^2 + \alpha x + \beta x + \alpha\beta - c = 0$

45. If the roots of the equation $x^2 - ax - bx - cx + bc + ca = 0$ are equal ,then which one of the is correct ? **(2022-I)**

(a)$a + b + c = 0$ (b) $a - b + c = 0$ (c) $a + b - c = 0$ (d) $-a + b + c = 0$

46. Let $\alpha$ *and* $\beta$ $(\alpha > \beta)$ be the roots of the equation $x^2 - 8x + q = 0$. If $\alpha^2 - \beta^2 = 16$ ,then what is the value of $q$ ? **(2022-I)**

(a)$-15$ (b) $-10$ (c) $10$ (d) $15$

47. If $|z + \bar{z}| = |z - \bar{z}|$ , then the locus of $Z$ is**(2014-I)**
(a) A pair of straight line (b) A line
(c) A set of four straight lines (d)A circle

48. For a complex number z, the minimum value of |z| + |z−2| is
(a) 1 (b) 2 (c) 3 (d) none

49. If z is a complex number, then $z^2 + z^{-2} = 2$ represents

(a) a circle (b) a straight line (c) an ellipse (d)a hyperbola

50. What is $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}}$ equal to, where $n$ is a natural number and $i = \sqrt{-1}$(**2014-II**)
(a) 2 (b) 2 $i$ (c) $-2\,i$ (d) $i$

51. If 1, $w$, $w^2$ are the three cube roots of unity then $(3 + w^2 + w^4)^6$ is
(a) 64 (b) 729 (c) 2 (d) 0

52. If $z^2 - z + 1 = 0$, then $z^n - z^{-n}$, where $n$ is a multiple of 3 is

(a) $2\,(-1)^n$ (b) 0 (c) $(-1)^{n+1}$ (d) None

53. If $w$ is a cube root of unity then the expression $(1 - w)(1 - w^2)(1 + w^4)(1 + w^8)$ is equal to

(a) 0 (b) 3 (c) 1 (d) 2

54. If $w$ is a cube root of unity, then$\frac{1+2w+3w^2}{2+3w+w^2} + \frac{2+3w+w^2}{3+w+2w^2}$is equal to

(a) -1 (b) 2w (c) 0 (d) -2w

55. For a positive integer $n$, the expression $(1 - i)^n(1 - \frac{1}{i})^n$ is

(a) 0 (b) $2i^n$ (c) $2^n$ (d) $4^n$

56. The value of $(-1 + \sqrt{-3})^{62} + (-1 - \sqrt{-3})^{62}$ is

(a) $2^{62}$ (b) $2^{64}$ (c)$-2^{62}$ (d) 0

57. If $\alpha, \beta$ are the roots of the equation $x^2 + x + 1 = 0$ and $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of the equation

$x^2 + px + q = 0$then$p$ is

(a) -1 (b) 1 (c) -2 (d) 2

58. If a,b,c are in G.P. and $z = \frac{a+ib}{c-ib}$, then z is

(a) $\frac{ib}{c}$ (b) $\frac{ic}{b}$ (c) $\frac{ia}{c}$ (d) 0

59. If $Z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|Z|$ is equal to

(a) 1 (b) 3 (c) 0 (d) 2

60. If $1, w, w^2$ are the cube roots of unity, then the value of

$(1+w)(1+w^2)(1+w^4)(1+w^8)$ is **(2015-I)**

(a) $-1$ (b) 0 (c) 1 (d) 2

61. What is the real part of $(\sin x + i\cos x)^3$ $where\ i = \sqrt{-1}$ ?**(2015-I)**

(a) $-\cos 3x$ (b) $-\sin 3x$ (c) $\sin 3x$ (d) $\cos 3x$

62. If $\alpha, \alpha^2$ be the roots of $x^2 + x + 1 = 0$, then the equation whose roots are $\alpha^{31}, \alpha^{62}$ is

(a) $x^2 - x + 1 = 0$ (b) $x^2 + x - 1 = 0$

(c) $x^2 + x + 1 = 0$ (d) $x^{60} + x^{30} + 1 = 0$

63. What is the argument of the complex number $(-1-i)$ where $i = \sqrt{-1}$ ?**(2013-I)**

(a) $\frac{5\pi}{4}$ (b) $-\frac{5\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) None

64. If $z = 1 + i\ tan\alpha,\ \pi < \alpha < \frac{3\pi}{2}$, then $|-5iz|$ is

(a) $5\cos\alpha$ (b) $\frac{-5}{\cos\alpha}$ (c) $\frac{5}{\cos\alpha}$ (d) $5\cot\alpha$

65. If $(a+ib)(c+id) = 3+5i$, then the value of $(a^2+b^2)(c^2+d^2)$ is

(a) $-4+30i$ (b) $4-30i$ (c) 29 (d) 34

66. If $w$ is a cube root of unity then $\tan\left[\left(\omega^{200} + \frac{1}{\omega^{200}}\right)\pi + \frac{\pi}{4}\right]$ is

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) 0 (d) none

67. What is the real part of $(sinx + icosx)^3$ where $i=\sqrt{-1}$ ?( **2015-I)**

(a) $-cos3x$ (b) $-sin3x$ (c) $sin3x$ (d) $cos3x$

68. If $\frac{z_2}{z_1}$ is purely imaginary, then $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\sqrt{13}$

69. Imaginary part of $\frac{1}{1+cos\theta-isin\vartheta}$ is

(a) $2tan\frac{\theta}{2}$ (b) $-\frac{1}{2}tan\frac{\theta}{2}$ (c) $-cosec\theta$ (d) $\frac{1}{2}\ tan\frac{\theta}{2}$

70. If $Z = \frac{1+2i}{2-i} - \frac{2-i}{1+2i}$, then what is the value of $z^2 + z\bar{z}$ ? $(i = \sqrt{-1})$**(2011-II)**

(a) 0 (b) $-1$ (c) 1 (d) 8

71. If $Z = \frac{cox\,\theta+\sin\theta}{\cos\theta-\sin\theta}, \frac{\pi}{4} < \theta < \frac{\pi}{2}$, then arg Z is

(a) $2\theta$ (b) $2\theta - \pi$ (c) $\pi + 2\theta$ (d) None

72. If $Z = x + iy$ satisfies $amp\,(Z-1) = amp\,(Z+3i)$, then the value of $(x-1):y$ is equal to

(a) 2 : 1 (b) 1 : 3 (c) −1 : 3 (d) None

73. The value of $(1+\omega^2+2\omega)^{3n} - (1+\omega+2\omega^2)^{3n}$ is

(a) 0 (b) 1 (c) $\omega$ (d) $\omega^2$

74. If $\left|Z-\frac{3}{Z}\right| = 2,$ then the greatest value of $|Z|$ is

(a) 1 (b) 2 (c) 3 (d) 4

75. If $Z_1 \& Z_2$ are two non – zero complex numbers such that $|Z_1+Z_2| = |Z_1|+|Z_2|$, then $arg\ Z_1 - arg\ Z_2$ is equal to (**AIEEE-2005**)

(a) $\pi/2$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

76. $z\bar{z}$ + (3–i)z + (3+i)$\bar{z}$ + 1 = 0 represent a circle with (2015-II)

(a) Centre (–3, –1) & radius 3 (b) Centre (–3,1) & radius 3

(c) Centre (–3, –1) & radius 4 (d) Centre (–3,1) & radius 4

77. If $\left|z-\frac{3}{z}\right|$= 2, then the greatest value of $|z|$ is

(a) 1 (b) 2 (c) 3 (d) 4

78. If w be the imaginary cube root of unity the value of $\frac{7+11\omega+13\omega^2}{13+7\omega+11\omega^2}+\frac{7+11\omega+13\omega^2}{11+13\omega+7\omega^2}$ is

(a) 2 (b) 3 (c) 0 (d) −1

79. Let $z, w$be two non-zero complex numbers. If $\overline{z+iw}$ = 0 and arg $(zw)$ = $\pi$, then arg z is

(a) $\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

80. If $\alpha, \beta$ be the roots of the equation $x^2+x+1=0$ then the equation whose roots are $\frac{\alpha}{\beta}\&\frac{\beta}{\alpha}$ is

(a) $x^2+x+1=0$ (b) $x^2-x+1=0$

(c) $x^2-x-1=0$ (d) $x^2+x-1=0$

81. If $\frac{(a+i)^2}{2a-1} = p+iq, then\ p^2+q^2$ is

(a) $\frac{(a^2+1)^2}{2a^2+1}$ (b) $\frac{(a^2+1)^2}{2a^2-1}$ (c) $\frac{(a^2+1)^2}{(2a^2-1)^2}$ (d) None

82. If $\alpha+i\beta = tan^{-1}z, when\ z = x+iy\ \&\ \alpha\ is\ a\ constant, then$

(a) $x^2 + y^2 + 2x\, cot2\alpha = 1$ (b) $(x^2 + y^2)\, cot2\alpha = 1 + x$

(c) $x^2 + y^2 + 2y\, tan\, 2\alpha = 1$ (d) $x^2 + y^2 + 2x\, sin\alpha = 1$

83. $|z_1 + z_2| = |z_1| + |z_2|$ is possible if

(a) $z_2 = \overline{z_1}$ (b) $z_2 = \frac{1}{z_1}$ (c) $argz_1 = argz_2$ (d)$|z_1| = |z_2|$

84. Let $z = x + iy$ *where* $x, y$ *are real variables and* $i = \sqrt{-1}$.*If* $|2z - 1| = |z - 2|$, then the point $z$ describes (**2014-I)**

(a) a circle (b) an ellipse (c) a hyperbola (d) a parabola.

85.The curve represented by $\text{Im}(z^2) = k$ where k is a non zero real number ,is

(a) a pair of straight lines (b) an ellipse (c) a parabola (d) a hyperbola

86.consider the following in respect of a complex numberZ : (2021-I)

1. $\overline{(Z^{-1})}=(\bar{Z})^{-1}$
2. $ZZ^{-1} = |Z|^2$

Which of the above is/are correct?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

87. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$ ,where $n \in N$ *and* $i = \sqrt{-1}$ (2021-I)

(a) -1 (b 1 (c) $\sqrt{2}$ (d) 2

88. If $z$ is a complex number of unit modulus and argument $\theta$, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals.

(a) $-\theta$ (b) $\frac{\pi}{2} - \theta$ (c) $\theta$ (d) $\pi - \theta$

89.What is the principal argument of $\frac{1}{1+i}$ where $i = \sqrt{-1}$ (2022-I)

(a) $-\frac{3\pi}{4}$ (b $-\frac{\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

90. What is the modulus of $\left(\frac{\sqrt{-3}}{2} - \frac{1}{2}\right)^{200}$ ? (2022-I)

(a) $\frac{1}{4}$ (b $\frac{1}{2}$ (c) 1 (d)$2^{200}$

91. How many words can be formed using all the letters of the word 'NATION' so that all the three vowels should never come together ?(**2015-I**)

(a) 354 (b) 348 (c) 288 (d) None

92.Consider the expansion 0f $(1+x)^n$. Let $p, q\ , r\ and\ s$ be the coefficients of first,second ,$nth\ and\ (n+1)th$ terms respectively .What is $(ps+qr)$ equal to ? **(2021-II)**

(a) $1+n$ (b) $1+2n^2$ (c) $1+n^2$ (d) $1+4n$

93.Consider the digits 3,5,7,9 .What is the number of 5-digit numbers formed by these digits in which each of these four digits appears ? **(2021-II)**

(a) 240 (b) 180 (c) 120 (d) 60

94. Let S=$\{2,3,4,5,6,7,9\}$ .How many different 3-digit number (with all digits different) from S can be made which are less than 500 ? **(2021-II)**

(a) 30 (b) 49 (c) 90 (d) 147

95. How many 4-letter words (with or without meaning) containing two vowels can be constructed using only the letters (without repetition) of the word LUCKNOW.? **(2021-II)**

(a) 240 (b) 200 (c) 150 (d) 120

96. If $C(n,4),\ C(n,5)\ \ and\ C(n,6)$ are in AP ,then what is the value of $n$? **(2021-II)**

(a) 7 (b) 8 (c) 9 (d) 10

97. How many terms are there in the expansion of $\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}+2\right)^{21}$

where $\neq 0, b \neq 0$ ? **(2021-II)**

(a) 21 (b) 22 (c) 42 (d) 43

98. How many permutations are there of the letters of the word TIGER in which the vowels should not occupy the even positions ? **(2022-I)**

(a) 72 (b) 36 (c) 18 (d) 12

99. What is the value of

$2(2\times 1)+3(3\times 2\times 1)+4(4\times 3\times 2\times 1)+5(\times 4\times 3\times 2\times 1)$

$+\cdots ...+9(9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1)$ ? **(2022-I)**

(a)11! (b) 10! (c) 10+10! (d)11+10!

100.Consider the following statements : **(2022-I)**

1. $\frac{n!}{3!}$ is divisible by 6, where $n > 3$

2. $\frac{n!}{3!} + 3$ is divisible by 7, where $n > 3$

Which of the above statements is/ are correct ?

(a)1 only (b) 2 only (c) both 1 and 2 (d)neither 1 nor 2

101. In how many ways can a team of 5 players be selected out of 9 players so as to exclude two particular players ? **(2022-I)**

(a)14 (b) 21 (c) 35 (d) 42

102. In the expansion of $\left(x + \frac{1}{x}\right)^{2n}$, what is the $(n+1)th$ term from the end (when arranged in descending power of $x$) ? **(2022-I)**

(a) C$(2n, n)x$ (b) C$(2n, n-1)x$ (c) C$(2n, n)$ (d) C$(2n, n-1)$

103.How many terms are there in the expansion of $\left(1 + \frac{2}{x}\right)^9 \left(1 - \frac{2}{x}\right)^9$? **(2022-I)**

(a)9 (b) 10 (c) 19 (d) 20

104. Consider the following statements in respect of the expansion of $(x + y)^{10}$: **(2022-I)**

1. Among all the coefficients of the terms ,the coefficient of the 6th term has the highest value.
2. The coefficient of the 3rd term is equal to coefficient of th 9th term.

Which of the above statements is/ are correct ?

(a)1 only (b) 2 only (c) both 1 and 2 (d)neither 1 nor 2

105. If C$(3n, 2n) = C(3n, 2n-7)$, then what is the value of C$(n, n-5)$ ? **(2022-I)**

(a)42 (b) 35 (c) 28 (d) 21

106. What is the value of
$C(51,21) - C(51,22) + C(51,23) - C(51,24) + C(51,25) - C(51,26) + C(51,27) - C(51,28) + C(51,29) - C(51,30)$ ? **(2022-I)**

(a) $C(51,25)$ (b) $C(51,27)$ (c)$C(51,51) - C(51,0)$ (d) $C(51,25) - C(51,27)$

107. How many odd numbers between 300 and 400 are there in which none of the digits is repeated ? **(2022-I)**

(a)32 (b) 36 (c) 40 (d) 45

108. How many four digit natural numbers are there such that all of the digits are odd ? **(2022-II)**

(a) 625 (b) 400 (c) 196 (d) 120

109. What is $\sum_{r=0}^{n} 2^r C(n, r)$ equal to? **(2022-II)**

(a) $2^n$ (b) $3^n$ (c) $2^{2n}$ (d) $3^{2n}$

110.If different permutations of the letters of the word MATHEMATICS are listed as in a dictionary ,how many words with or without meaning are there in the list before the first word that start with C ? **(2022-II)**

(a) 302400 (b) 403600 (c) 907200 (d) 1814400

Consider the following for the next three (03) items that follow : **(2022-II)**

Consider the word 'QUESTION':

111. How many 4-letters word each of two vowels and two consonants with or without meaning ,can be formed ?

(a) 36 (b) 144 (c) 576 (d) 864

112. How many 8-letter words with or without meaning, can be formed such that consonants and vowels occupy alternate positions ?

(a) 288 (b) 576 (c) 1152 (d) 2304

113. How many 8-letter words with or without meaning, can be formed so that all consonants are together ?

(a) 5760 (b) 2880 (c) 1440 (d) 720

Consider the following for the next three (03) items that follow : **(2022-II)**

Consider the binomial expansion of $(p + qx)^9$:

114. What is the value of $q$ if the coefficents of $x^3$ and $x^6$ are equal ?

(a) *p* (b) 9*p* (c) $\frac{1}{p}$ (d) $p^2$

115. What is the ratio of the coefficients of middle terms in the expansion (when expanded in ascending powers of *x* ) ?

(a) *p* (b) p/q (c) 4*p*/5*p* (d)1/(*pq*)

116. Under what conditionthe coefficients of $x^2$and $x^4$are equal?

(a) $p:q = 7:2$ (b) $p^2:q^2 = 7:2$ (c) $p:q = 2:7$ (d) $p^2:q^2 = 2:7$

117. A box contains 2 white balls ,3 black balls and 4 red balls. What is the number of ways of drawing 3 balls from the box with at least one black ball ? **(2022-II)**

(a) 84 (b) 72 (c) 64 (d) 48

118. What is the sum to $n$ terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots\ldots$ ?(**2015-II**)

(a) $\frac{n(n-1)}{\sqrt{2}}$ (b) $\sqrt{2}\, n\,(n+1)$ (c) $\frac{n(n+1)}{\sqrt{2}}$ (d) $\frac{n(n-1)}{2}$

119. If $\frac{1}{ab+ac}, \frac{1}{bc+ba}, \frac{1}{ca+cb}$ are in H.P, then a,b,c are in (**2016-II**)

(a) A.P (b) G.P (c) H.P (d) none

120. The middle term of an arithmetic progression consisting of n number of terms is m, the sum of the terms of the series is

(a) m + n (b) mn (c) m – n (d)

121.In an AP, the pth term is –q and the (p+q)th term is 0. Then the $q^{th}$ term is

(a) –p (b) p (c) p + q (d) p – q

122. The largest term common to the sequences 1,11,21,31,…. to 100 terms and 31,36,41,46… to 100 terms is

(a) 381 (b) 471 (c) 281 (d) None

123. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\ldots$ to $\infty = \frac{\pi^4}{90}$, then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots\ldots$ $\infty$ is equal to

(a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^4}{45}$ (c) $\frac{89\pi^4}{90}$ (d) None

124. if $(1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{128}) = \sum_{r=0}^{n} x^r$ then n is

(a) 255 (b) 127 (c) 60 (d) None

125. The sum of the series $\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\ldots\ldots\propto$ is equal to

(a) 1 (b) 0 (c) –1 (d) 2

126. If $x^2+9y^2+25z^2 = xyz\left(\frac{15}{x}+\frac{5}{y}+\frac{3}{z}\right)$ *then x, y, z are in*

(a) A.P (b) G.P (c) H.P (d) None.

127. If AM and GM of two numbers are 10 and 8 respectively ,then one number exceed the other number by

(a) 8 (b) 10 (c) 12 (d) 2

128.The harmonic mean H of two numbers is 4 and the arithmetic mean A and geometric mean G satisfy the equation $2A + G^2 = 27$ .The two numbers are

(a) 6,3 (b) 9,5 (c) 12,7 (d) 3,1

129.Four numbers are in AP .The sum of first and last term is 8 and the product of both middle term is 5. The last number of the series is

(a) 4 (b) 3 (c) 2 (d) 1

130. If n arithmetic means are inserted between 2 and 38. Then the sum of resulting series is obtained as 200 ,then what is the value of n ?

(a) 6 (b) 8 (c) 9 (d) 10

131. If $x, y, z$ are in GP, then which of the following is/are correct ? **(2021-II)**

1. $ln(3x)$ , $ln(3y), ln(3z)$ are in AP .

2. $xyz + \ln(x), xyz + \ln(y), xyz + \ln(z)$ are in HP

Select the correct answer using the code given below .

(a) 1 only (b) 2 only (c) both 1 and 2 (d) neither 1 nor 2

132.If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in HP, then which of the following is/are correct ? **(2021-II)**

1. $a, b, c$ are in AP

2.$(b+c)^2, (c+a)^2, (a+b)^2$ are in GP.

Select the correct answer using the code given below.

(a) 1only (b) 2only (c) both 1 and 2 . (d) neither 1 nor 2.

133. Let $S_k$ denote the sum of first k terms of an AP . What is $\frac{S_{30}}{S_{20}-S_{10}}$ equal to ? **(2021-II)**

(a) 1 (b) 2 (c) 3 (d) 4

134. If $a, b, c$ are in GP where $a > 0, b > 0, c > 0$, then which of the following are correct ? **(2022-I)**

1. $a^2, b^2, c^2$ are in GP.

2. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in GP.

3. $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in GP.

Select the correct answer using the code given below :

(a) 1 and 2only (b) 2 and 3 only (c) 1 and 3 only. (d) 1,2 and3

135. If $\frac{a+b}{2}, b, \frac{b+c}{2}$ are in HP ,then which one of the following is correct ? **(2022-I)**

(a) $a, b, c$ are in AP

(b) $a, b, c$ are in GP

(c) $a+b, b+c, c+a$ are in GP

(d) $a+b, b+c, c+a$ are in AP

136. If the sum of the first 9 terms of an AP is equal to sum of the first 11 terms, then what is the sum of the sum of the first 20 terms ? **(2022-I)**

(a) 20 (b) 10 (c) 2 (d) 0

137. If the $5^{th}$ term of an AP is $\frac{1}{10}$ and its $10^{th}$ term is $\frac{1}{5}$ , then what is the sum of first 50 terms ? **(2022-I)**

(a) 25 (b) 25.5 (c) 26 (d) 26.5

Consider the following for the next three items that follow: **(2022-II)**

Let $P$ be the sum of first $n$ positive terms of an increasing arithmetic progression $A$. Let $Q$ be the sum of first $n$ positive terms of another increasing arithmetic progression $B$ .Let $P:Q = (5n+4):(9n+6)$.

96. What is the ratio of the first term of $A$ to that of $B$ ?

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{3}{5}$

138. What is the ratio of their $10^{th}$ terms ?

(a) $\frac{11}{29}$ (b) $\frac{22}{49}$ (c) $\frac{33}{59}$ (d) $\frac{44}{69}$

139.If d is the common difference of A and D is the common difference of B ,then which of the following is always correct ?

(a) $D > d$ (b) $D < d$ (c) $7D > 12d$ (d) none of the above.

140. What is $\frac{cos7x - cos3x}{sin7x - 2sin5x + sin3x}$ equal to ? **(2014-II)**

(a) $tanx$ (b) $cotx$ (c) $tan2x$ (d) $cot2x$

141. Find the value of cos $(\frac{29\pi}{3})$

(a) 1 (b) 0 (c) $\frac{\sqrt{3}}{2}$ (d) ½

142. From an Aeroplane above a straight road the angle of depression of two positions at a distance 20 m apart on the road are observed to be $30^0$& $45^0$. The height of the Aeroplane above the ground is **(2014-I)**

(a) $10\sqrt{3}m$ (b) $10(\sqrt{3}-1)m$ (c) $10(\sqrt{3}+1)m$ (d) $20m$

143. In any triangle ABC, a=18, b=24 & c=30. Then what is Sin C equal to ? **(2013-I)**

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

144. What is $tan(-585°)$ equal to ? **(2012-II)**

(a) 1 (b) -1 (c) $-\sqrt{2}$ (d) $-\sqrt{3}$

145. Consider the following statements: **(2014-I)**

1. Value of $sin\theta$ oscillates between -1 & 1.
2. Value of $cos\theta$ oscillates between 0 & 1.

Which of the above statements is / are correct?

(a) 1 only (b) 2 only (c) both 1 and 2 (d) Neither 1 nor 2

146. In a $\Delta PQR$, if $3SinP + 4CosQ = 6 \; and \; 4SinQ + 3CosP = 1$,then the angle $R$ is equal to**(AIEEE-2012)**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

146. In a triangle ABC, $cosA = \frac{sinB}{2\, sinC}$ , then the triangle is

(a) equilateral (b) isosceles (c) right angled (d) None

147. In $\Delta ABC$, if $a = 2, \; b = 4 \; and \; \angle c = 60°$ then $\angle A$ &$\angle B$ are respectively:

(a) $90^0$ , $30^0$  (b) $60^0$, $60^0$  (c) $30^0$, $90^0$  (d) $60^0$, $45^0$

148. Two poles of equal height stand on either side of a 100 m wide road. At a point between the poles the angles of elevation of the topes of the poles are $30^0$& $60^0$. The height of each pole is

(a) 25 m  (b) $25\sqrt{3}m$  (c) $\frac{100}{\sqrt{3}}m$  (d) None

149. The value of $sin12° sin48° sin54°$ is equal to :

(a) $\frac{1}{16}$  (b) $\frac{1}{32}$  (c) $\frac{1}{8}$  (d) $\frac{1}{4}$

150. The valuel of $\cos^2 A(3-4cos^2A)^2 + sin^2A(3-4sin^2A)^2$ is

(a) $sin4A$  (b) $cos4A$  (c) 1  (d) None

151. What is the value of $cos36°$ ? **(2014-I)**

(a) $\frac{\sqrt{5}-1}{4}$  (b) $\frac{\sqrt{5}+1}{4}$  (c) $\frac{\sqrt{10+2\sqrt{5}}}{4}$  (d) $\frac{\sqrt{10-2\sqrt{5}}}{4}$

152. The range of the function $f(x) = \frac{1}{2-cos3x}$ is

(a) $(-2, \alpha)$  (b) $[-2 ,3]$  (c) $\left[\frac{1}{3}, 1\right]$  (d) $\left(\frac{1}{2} , 1\right)$

153. If $sinx + siny = \frac{1}{2}$&$cosx + cosy = 1$, then $\tan(x+y)$ is

(a) $-\frac{4}{3}$  (b) $\frac{4}{3}$  (c) $\frac{8}{3}$  (d) $-\frac{8}{3}$

154. The angle of elevation of the top of a tower standing on a horizental place from two points on a line passing through the foot of the tower at distances 49 m and 36 m are $43^0$& $47\ ^0$ respectively. What is the height of the tower ? **(2015-I)**

(a) $40\ m$  (b) $42\ m$  (c) $45\ m$  (d) $47\ m$

155. The value of $\left(1+\cos\frac{\pi}{6}\right)\left(1+cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{7\pi}{6}\right)$ is

(a) $\frac{3}{16}$  (b) $\frac{3}{8}$  (c) $\frac{3}{4}$  (d) $\frac{1}{2}$

156. If $0 \le x \le \pi, 0 \le y \le \pi$ and $\sin x + \sin y = 2$, then the value of $\cos(x+y)$ is

(a) $-1$  (b) 1  (c) 0  (d) None

157. The value of $\cos 15° \ \cos 7\frac{1}{2}^{\circ} \ \sin 7\frac{1}{2}^{\circ}$ is

(a) $\frac{1}{2}$  (b) $\frac{1}{8}$  (c) $\frac{1}{4}$  (d) $\frac{1}{6}$

158. $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ is equal to

(a) $\sec\frac{x}{2}$  (b) $\sec x$  (c) $cosec\ x$  (d) 1

159. If $p = \sin^6 x + \cos^6 x$, then

(a) $\frac{3}{4} \le p \le 1$  (b) $\frac{1}{4} \le p \le 1$  (c) $\frac{3}{8} \le p \le \frac{1}{2}$  (d) $None$

160. If in a $\Delta ABC, a\cos^2\left(\frac{C}{2}\right) + c\ \cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then sides $a, b, c$ (**AIEEE-2003**)

(a) are in A.P  (b) are in G.P  (c) are in H.P  (d) satisfy at $b = c$

161. The maximum and minimum value of $6\ sinx\ cosx + 4\ cos2x$, are respectively
(a) $3, -3$ (b) $-5, 5$ (c) $5, -5$ (d) None

162. If $sinx + sin^2x = 1$, then $cos^6x + cos^{12}x + 3cos^{10}x + 3cos^8x$ is equal to
(a) 1 (b) $cos^3x\ sin^3x$ (c) 0 (d) $\alpha$

For the next two(2) items that follow: **(2015-I)**

Let $\alpha$ be the root of the equation $25\cos^2\theta + 5\cos\theta - 12 = 0\ where\ \frac{\pi}{2} < \alpha < \pi$

163. What is $tan\alpha$ equal to ?

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{4}{3}$ (d) $-\frac{4}{5}$

164. What is $sin2\alpha$ equal to ?

(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$ (c) $-\frac{5}{12}$ (d) $-\frac{21}{25}$

165. From the top of a lighthouse 70m high with its base at sea level, the angle of depression of a boat is 15°. The distance of the boat from the foot of the lighthouse is**(2012-II)**
(a) $70(2-\sqrt{3})$m (b) $70(2+\sqrt{3})$m (c) $70(3-\sqrt{3})$m (d) $70(3+\sqrt{3})$m

For the next two (2) items that follow:- **(2016-I)**
Given that $tan\ \alpha$ and $tan\ \beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$

166. What is $tan\ (\alpha + \beta)$ equal to ?

(a) $b\ (c-1)$ (b) $c\ (b-1)$ (c) $c\ (b-1)^{-1}$ (d) $b\ (c-1)^{-1}$

167. What is $sin\ (\alpha + \beta) sec\ \alpha \sec\ \beta$ equal to ?

(a) $b$ (b) $-b$ (c) $c$ (d) $-c$

168. The value of $\frac{\sqrt{3}}{\sin 15°} - \frac{1}{\cos 15°}$ is equal to ?

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

169. The value of $sin^2\frac{\pi}{8} + sin^2\frac{3\pi}{8} + sin^2\frac{5\pi}{8} + sin^2\frac{7\pi}{8}$is

(a) $1/_8$ (b) $1/_4$ (c) $1/_2$ (d) 2

170. The value of $\tan\frac{\pi}{8}$ is equal to
(a) $1/_2$ (b) $\sqrt{2} + 1$ (c) $1 - \sqrt{2}$ (d) $\sqrt{2} - 1$

171. The minimum value of $atan^2x + b\ cot^2x$ equals the maximum value of $asin^2x + b\ cos^2x$, where $a > b > 0$ , then

(a) $a = b$ (b) $a = 2b$ (c) $a = 3b$ (d) $a = 4b$

172. The value of $\cos 15° \cos\left(7\frac{1}{2}\right)° sin\left(7\frac{1}{2}\right)°$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

173. If in a $\triangle ABC, sin^2A + sin^2B + sin^2C = 2$, then the triangle is always

(a) isoscelese triangle (b) right angled (c)acute angled (d) obtuse angled

174. The value of $cos\frac{2\pi}{15}.cos\frac{4\pi}{15}.cos\frac{8\pi}{15}.cos\frac{16\pi}{15}$ is equal to

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$ (c) $\frac{1}{64}$ (d) $\frac{1}{8}$

175. The number of integral value of $K$ for which the equation $7\,cosx + 5\sin x = 2k + 1$ has a solution is

(a) 4 (b) 8 (c) 10 (d) 12

176. If $x$ = sin70°. sin50° and y = cos60°. cos80°, then what is $x$y equal to? **(2016-II)**

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

177. If Sin$\theta$ = $x + \frac{a}{x} \forall x \in$ R - {0}, then

(a) a$\geq \frac{1}{4}$ (b) a$\geq \frac{1}{2}$ (c) a $\leq \frac{1}{4}$ (d) a $\leq \frac{1}{2}$

178. The value of $sin^2\frac{\pi}{8} + sin^2\frac{3\pi}{8} + sin^2\frac{5\pi}{8} + sin^2\frac{7\pi}{8}$ is

(a) 2 (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) 1

179. if y = $\frac{1}{2}sin^2\theta + \frac{1}{3}cos^2\theta$, then

(a) $\frac{1}{3} \leq y \leq \frac{1}{2}$ (b) $y \geq \frac{1}{2}$ (c) $2 \leq y \leq 3$ (d) $-\frac{\sqrt{13}}{6} \leq y \leq \frac{\sqrt{13}}{6}$

180. The equation $\sqrt{3}$ sinx + cosx = 4 has

(a) Infinitely many solution (b) No solution

(c) Two Solution (d) Only one solution

181. In a triangle ABC, if $acos^2\frac{C}{2} + ccos^2\frac{A}{2} = \frac{3b}{2}$ then the sides a,b,c

(a) Satisfy a + b = c (b) are in AP (c) are in GP (d) are in HP

182. The value of $\frac{\sqrt{3}}{\text{Sin}15°} - \frac{1}{\text{cos}15°}$ is equal to

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

183. If $x\text{cos}\theta + ysin\theta = z$, then what is the value of $(xsin\theta - ycos\theta)^2$? **(2016-II)**

(a) $x^2 + y^2 - z^2$ (b) $x^2 - y^2 - z^2$

(c) $x^2 - y^2 + z^2$ (d) $x^2 + y^2 + z^2$

184. What is $\frac{cos\theta}{1-tan\theta} + \frac{sin\theta}{1-cot\theta}$ equal to? **(2015-I)**

(a) $sin\theta - cos\theta$ (b) $sin\theta + cos\theta$ (c) $2sin\theta$ (d) 2

185. If $x = sin70°. sin50°$ $and\ y = cos60°. cos80°$ $then\ what\ is\ xy\ equal\ to$ ? **(2016-II)**

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

186. If $x^2 - 2x + sin^2\alpha = 0$, then

(a) $x \in [-1, 1]$ (b) $x \in [0, 2]$ (c) $x \in [-2, 2]$ (d) $x \in [1, 2]$

187. If $0 < x < \frac{\pi}{2}$, then the minimum value of $(sinx + cosx + cosec2x)^3\ is$

(a) 27 (b) 13.5 (c) 6.75 (d) None

For the next two items that follow: **-(2016-I)**

Consider the equation $ksinx + cos2x = 2k - 7$

188 If the equation possesses solution, then what is the minimum value of k ?

(a) 1 (b) 2 (c) 4 (d) 6

189. If the equation possesses solution, then what is the maximum value of k ?

(a) 1 (b) 2 (c) 4 (d) 6

190. If $A = (cos12° - cos36°)(sin96° + sin24°)$ & $B = (sin60° - sin12°)(cos48° - cos72°)$ then what is $\frac{A}{B}$ equal to ? **(2016-I)**

(a) $-1$ (b) 0 (c) 1 (d) 2

191. What is $\frac{1-tan2°cot62°}{tan152°-cot88°}$ equal to? **(2016-II)**

(a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\sqrt{2} - 1$ (d) $1 - \sqrt{2}$

192. If $3sin\theta + 5cos\theta = 5$, then the value of $5sin\theta - 3cos\theta$ is equal to

(a) 5 (b) 3 (c) 4 (d) None

193. If $x = ycos\frac{2\pi}{3} = zcos\frac{4\pi}{3}, then\ xy + yz + zx$ is equal to

(a) $-1$ (b) 0 (c) 1 (d) 2

194. If $\frac{3\pi}{4} < \alpha < \pi, then$ $\sqrt{2cot\alpha + \frac{1}{sin^2\alpha}}$ is equal to

(a) $1 + cot\alpha$ (b) $-(1 + cot\alpha)$ (c) $1 - cot\alpha$ (d) $-1 + cot\alpha$

195. If in a $\Delta ABC, (a + b + c)(b + c - a) = \lambda bc$ then

(a) $\lambda < 0$ (b) $\lambda > 4$ (c) $\lambda > 0$ (d) $0 \leq \lambda \leq 4$

196. In a triangle ABC if the angles A, B, C are in A.P, then which one of the following is correct? **(2012-I)**

(a) $c = a + b$ (b) $c^2 = a^2 + b^2 - ab$

(c) $a^2 = b^2 + c^2 - bc$ (d) $b^2 = a^2 + c^2 - ac$

For the next two items that follow:**(2016-I**)

Consider the lines: y = 3x, y = 6x and y = 9

197. What is the area of the triangle formed by these lines ?

(a) $\frac{27}{4}$ square units (b) $\frac{27}{2}$ square units (c) $\frac{19}{4}$ square units (d) $\frac{19}{2}$ square units

198. The centroid of the triangle is at which one of the following points?

(a) $(3, 6)$ (b) $\left(\frac{3}{2}, 6\right)$ (c) $(3,3)$ (d) $\left(\frac{3}{2}, 9\right)$

199. If the three points (3q,0), (0,3p) and (1,1) are collinear then which one of the following is true?

(a) $\frac{1}{p} + \frac{1}{q} = 1$ (b) $\frac{3}{p} + \frac{1}{q} = 1$ (c) $\frac{1}{p} + \frac{1}{q} = 3$ (d) $\frac{1}{p} + \frac{3}{q} = 1$

200. An equilateral triangle has one vertex at (0,0) and another at (3, $\sqrt{3}$). What are the co-ordinates of the third vertex?**(2016-I**)

(a) (0, $2\sqrt{3}$) only (b) (3,$-\sqrt{3}$) only

(c) (0, $2\sqrt{3}$) or (3, $-\sqrt{3}$) (d) Neither (0, $2\sqrt{3}$) nor (3, $-\sqrt{3}$)

201. What is the equation of the right bisector of the line segment joining (1,1) & (2,3)?**(2016-I**)

(a) 2 x + 4y – 11 = 0 (b) 2 x – 4y – 5 = 0

(c) 2 x – 4y – 11 = 0 (d) x – y +1 = 0

202. The eccentricity of the hyperbola $x^2 - 3y^2 + 1 = 0$ is**(2016-II**)

(a) 3 (b) 2 (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

203. The image of the origin with respect to the line 4x + 3y = 25 is

(a) (4,3) (b) (3,4) (c) (6,8) (d) (8,6)

204. The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150° with the positive direction of y-axis. The equation of the line is

(a) $\sqrt{3}x+y = 14$ (b) $\sqrt{3}x - y = 14$ (c) $3x - y = 14$ (d) None

205. If the sum of the reciprocals of the intercepts, cuts off by a straight line is a constant k, then the line passes through the fixed point

(a) (k,k) (b) $\left(\frac{1}{k}, \frac{1}{k}\right)$ (c) (k, − k) (d) (− k, k)

206. If e and $e'$ are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ and the hyperbola $5x^2 - 4y^2 = 45$ respectively, then e $e'$ is equal to

(a) 9 (b) 4 (c) 5 (d) 1

207. The equation to the hyperbola having its eccentricity 2 and the distance between its foci is 8 is

(a) $\frac{x^2}{12} - \frac{y^2}{4} = 1$ (b) $\frac{x^2}{4} - \frac{y^2}{12} = 1$ (c) $\frac{x^2}{8} - \frac{y^2}{2} = 1$ (c) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

For the next three items that follow: **(2016-I)**

Consider a parallelogram whose vertices are $A(1,2), B(4,y), c(x,6) \& D(3,5)$ taken in order.

208. What is the value of $AC^2 - BD^2$?

(a) 25 (b) 30 (c) 36 (d) 40

209. What is the point of intersection of the diagonals?

(a) $\left(\frac{7}{2}, 4\right)$ (b) (3, 4) (c) $\left(\frac{7}{2}, 5\right)$ (d) (3, 5)

210. What is the area of the parallelogram?

(a) $\frac{7}{2}$ sq unit (b) 4 sq unit (c) $\frac{11}{2}$ sq unit (d) 7 sq unit

211. If the ellipse $9x^2 + 16y^2 = 144$ intercepts the line $3x + 4y = 12,$ then what is the length of the chord so formed ?**(2016-I)**

(a) 5 units (b) 6 units (c) 8 units (d) 10 units

212. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change of $\propto$ ?

(a) abscissae of vertices (b) eccentricity

(c) abscissae of foci (d) directrix

213. The radius of the circle $x^2 + y^2 + x + c = 0$ passing through the origin is**(2013-II)**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

214. What is the perpendicular distance between the parallel lines $3x + 4y = 9$ & $9x + 12y + 28 = 0$ ?**(2012-II**)

(a) $\frac{7}{3}$ units (b) $\frac{8}{3}$ units (c) $\frac{10}{3}$ units (d) $\frac{11}{3}$ units

215. The centre of the circle $(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$ is (2020- I and II)

(a) $(2a, 2c)$ (b) $(2b, 2d)$ (c) $(a + b, c + d)$ (d) $(a - b, c - d)$)

216. Let $P\ (x, y)$ be any point on the ellipse $25x^2 + 16y^2 = 400$ .If $Q(0,3)$ and $R\ (0, -3)$ are two points ,then what is $(PQ\ +\ PR)$ equal to? (2020-I and II)

(a) 12 (b) 10 (c) 8 (d) 6

217. The point $(1, -1)$ is one of the vertices of a square .If $3x + 2y = 5$ is the equation of one diagonal of the square ,then what is the equation of the other diagonal?

(a) 3x-2y = 5 (b) 2x – 3y = 1 (c) 2x – 3y = 5 (d) 2x + 3y = -1

218 In the parabola ,$y^2 = x$ , what is the length of the chord passing through the vertex and inclined to the x –axis at an angle $\theta$ ?(2020-I and II)

(a) $\sin\theta.\sec^2\theta$ (b) $\cos\theta.\ cosec^2\,\theta$ (c)$\cot\theta.\sec^2\theta$ (d)$\tan\theta.cosec^2\theta$

219. The measure which takes into account all the data items is

(a) mean (b) median (c) mode (d) None

220. Variance is always independent of the change of**(2013-I**)

(a) origin but not scale(b) Scale only (c) both origin and scale (d) None

221. The root mean square deviation is least when deviations are measured from

(a) mean (b) mode (c) median (d) origin

222. The standard deviation of the set of first n natural number is

(a) $\frac{\sqrt{n^2-1}}{4n}$ (b) $\frac{\sqrt{n^2+1}}{4n}$ (c) $\frac{\sqrt{n^2+1}}{2n}$ (d) $\sqrt{\frac{n^2-1}{12}}$

223. If the mean of the set of numbers $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$ is $\bar{x}$, then the mean of the numbers $x_i + 2i, 1 \leq i \leq n$ is

(a) $\bar{x} + 2n$ (b) $\bar{x} + 2$ (c) $\bar{x} + n + 1$ (d) $\bar{x} + n$

224. If the mean of a set of observation $x_1, x_2, \ldots\ldots\ldots\ldots, x_{10}$ is 20, then the mean of $x_1 + 4,\ x_2 + 8, \ldots\ldots\ldots, x_{10} + 40$ is

(a) 34 (b) 38 (c) 40 (d) 42

225. The median of distribution 83, 54, 78, 64, 90, 59, 67, 72, 70, 73, is

(a) 71 (b) 70 (c) 72 (d) None

For the next two items that follow:
The frequency distribution table is given here

| | | | | | |
|---|---|---|---|---|---|
| $x_i$ | 10 | 15 | 18 | 20 | 25 |
| $f_i$ | 3 | 2 | 5 | 8 | 2 |

226. Find the variance

(a) 19 (b) 17 (c) 18 (d) 16

227. Find the standard deviation

(a) 4.12 (b) 5.12 (c) 6.12 (d) 7.12

228. If the coefficient of variation of some observations is 60 and their standard deviation is 20, then their mean is

(a) 35 (b) 34 (c) 38.3 (d) 33.33

229. The mean and variance of 10 observations are given to be 4 & 2 respectively. If every observation is multiplied by 2, the mean & variance of the new series will be respectively (**2015-I**)

(a) 8 & 20 (b) 8 & 4 (c) 8 & 8 (d) 80 & 40

230. The mean deviation about mean of the set of numbers 7, 9, 24, 14 & 26 is

(a) 7.5 (b) 8 (c) 7.2 (d) 7

231. If all the natural numbers from 1 to 20 are multiplied by 3 then what is the vaiance of the resulting series ?(**2019-I**)

(a) 99.75 (b)199.75 (c) 299.25 (d) 399.25

232. The standard deviation of the numbers 31, 32, 33, …….. 46, 47 is

(a) $\sqrt{\frac{17}{12}}$ (b) $\sqrt{\frac{47^2-1}{12}}$ (c) $2\sqrt{6}$ (d) $4\sqrt{3}$

234. The standard deviation of a distribution is 30 and each item is raised by 3, then new S.D is

(a) 32 (b) 28 (c) 27 (d) None

235. For the data 3,5,1,6,5,9,5,2,8,6, the mean median and mode are $x, y, \& z$ respectively. Which one of the following is correct ?(**2016-I**)

(a) $x = y \neq z$ (b) $x \neq y = z$ (c) $x \neq y \neq z$ (d) $x = y = z$

236. A batsman in his 16th innings makes a score of 70 runs and thereby increases his average by 2 runs. If he had never been 'not out' , then his average after 16th innings is

(a) 36 (b) 40 (c) 38 (d) 42

237. The average monthly salary of workers in a factory is Rs 206. If the average monthly salary of males and females are Rs 210 & Rs 190 respectively, the parcentage of female employed in the factory is

(a) 10 % (b) 50 % (c) 30% (d) 20%

238. The standard deviation of 50 values of a variable $x$ is 15, if each value of the variable is divided by (−3), then the standard deviation of the new set of 50 values of $x$ will be

(a) 15 (b) −5 (c) 5 (d) −15

239. If the total number of observation is 20, $\sum x_i = 1000$ & $\sum x_i^2 = 84000$, then what is the variance of the distribution? (**2016-I)**

(a) 1500 (b) 1600 (c) 1700 (d) 1800

240. If r is correlation coefficient, then correct relation is

(a) $r > 1$ (b) $r \geq 1$ (c) $|r| \leq 1$ (d) $|r| \geq 1$

241. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is(**2015-I**)

(a) 28 (b) 30 (c) 35 (d) 38

242. The mean of the series $x_1, x_2, \ldots\ldots\ldots x_n$ is $\bar{X}$. If $x_2$ is replaced by $\lambda$, then what isthe new mean? (**2016-I**)

(a) $\bar{X} - x_2 + \lambda$ (b) $\frac{\bar{X} - x_2 - \lambda}{n}$

(c) $\frac{\bar{X} - x_2 + \lambda}{n}$ (d) $\frac{n\bar{X} - x_2 + \lambda}{n}$

243. The arithmetic mean of 1, 8, 27, 64, …….. upto n terms is given by(**2015-II**)

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$ (c) $\frac{n(n+1)^2}{4}$ (d) $\frac{n^2(n+1)^2}{4}$

244. The variance of numbers $x_1, x_2, \ldots\ldots x_n$ is V. Consider the following statements.

(1) If every $x_i$ is increased by 2, the variance of the new set of numbers is V.

(2) If the numbers $x_i$ is squared, the variance of the new set is $V^2$.

Which of the following statements is / are correct?(**2014-I**)

*(a) 1 only (b)2 only (c)both 1 & 2 (d)Neither 1 nor 2*

245. If the variance of the data 2, 4, 5, 6, 17 is V, then what is the variance of the data 4, 8, 10, 12, 34? (**2011-II**)

(a) V (b) 4V (c) $V^2$ (d) 2V

246. If the correlation coefficient between two variables $x$&$y$ is $0.4$& regression coefficient of $x$ on $y$ is $0.2$, then the regression coefficient of $y$ on $x$ is

(a) 0.4 (b) $\pm 0.8$ (c) 0.8 (d) 0.6

247. The A.M of n numbers of a series is $\bar{X}$. If the sum of first (n −1) terms is k, then the $n^{th}$ number is

(a) $\bar{X} - k$ (b) $n\bar{X} - k$ (c) $\bar{X} - nk$ (d) $n\bar{X} - nk$

248.The G M of a set of observationsis computed as 10.The GM obtained when each observations $x_i$ is replaced by $3x_i^4$ is **(2021-I)**

(a) 810 (b) 900 (c)30000 (d) 81000

249.The coefficient of correlation is independent of **(2021-I)**

(a) change of scale only (b) change of origin only

(c) both change of scale change of origin (d) neither change of scale nor change of origin

250.The sum of deviations of n number of observations measured from 2.5 is 50. The sum of deviations of the same set of observations measured from 3.5 is -50. What is the value of n? (2020-I,II)

(a) 50 (b) 60 (c)80 (d) 100

251.If $\sum x_i = 20, \sum {x_i}^2 = 200$ and $n = 10$ for an observed variable x, then what is the coefficient of variation**?(2020-I,II)**

(a) 80 (b) 100 (c)150 (d) 200

252. If m is the geometric mean of $\left(\frac{y}{z}\right)^{\log(yz)}, \left(\frac{z}{x}\right)^{\log(zx)}, \left(\frac{x}{y}\right)^{\log(xy)}$ then what is the value of m ?

(a) 1 (b) 3 (c)6 (d) 9



Consider the following for the next two items which follow: (**2021-II**)

The mark obtained by 60 students in a certain subject out of 75 are given below :

| **Marks** | **Number of students** |
|---|---|
| 15 – 20 | 3 |
| 20 – 25 | 12 |
| 25–30 | X |
| 30 – 35 | 35 |
| 35 – 40 | Y |
| 40– 45 | 4 |

| | |
|---|---|
| 45-50 | |
| 50-55 | |
| 55-60 | |
| 60-65 | |

253. What is the median?

(a) 35 (b) 38 (c) 39 (d) 40

254. What is the mode?

(a) 27.27 (b) 27.73 (c) 27.93 (d) 28.27

Consider the following for the next two items which follow: (**2021-II**)

Two regression lines are given as $3x - 4y + 8 = 0$ and $4x - 3y - 1 = 0$

255 . Consider the following statements :

1. The regression line of y on x is $y = \frac{3}{4}x + 2$

2. The regression line of x on $y$ is $x = \frac{3}{4}y + \frac{1}{4}$

Which of the above statements is/ are correct?

(a) 1 only (b) 2 only (c) both 1 and 2 (d) neither 1 nor 2

256. Consider the following statements :

1. The coefficient of correlations r is $\frac{3}{4}$.

2. The means of x and y are 3 and 4 respectively .

Which of the above statements is/ are correct?

(a) 1 only (b) 2 only (c) both 1 and 2 (d) neither 1 nor 2

257. What is the mean of natural numbers contained in the interval [15,64] (**2021-II**)

(a) 36.8 (b) 38.3 (c) 39.5 (d) 40.3

258. For the set of numbers $x, x, x + 2, x + 3, x + 10$

where x is a natural number ,which of the following is/are correct? (**2021-II**)

1. Mean > Mode

2. Median > Mean

Select the correct answer using the code given below.

(a) 1 only (b) 2 only (c) both 1 and 2 (d) neither 1 nor 2

259. The mean of 10 observations is 5.5. If each observation is multiplied by 4 and subtracted from 44, then what is the new mean ? (**2021-II**)

(a) 20 (b) 22 (c) 34 (d) 44

260. If g is the geometric mean of 2,4,8,16,32,64,128,256,512,1024 ,then which one of the following is correct? (**2021-II**)

(a) $8 < g < 16$ (b) $16 < g < 32$ (c) $32 < g < 64$ (d) $g > 64$

261. If the harmonic mean of 60 and x is 48, then what is the value of x ? (**2021-II**)

(a) 32 (b) 36 (c) 40 (d) 44

262. What is the mean deviation of first 10 even natural numbers ? (**2021-II**)

(a) 5 (b) 5.5 (c) 10 (d)10.5

263. If $\sum_{i=1}^{10} x_i = 110$ and $\sum_{i=1}^{10} {x_i}^2 = 1540$ then what is the variance ? (**2021-II**)

(a) 22 (b) 33 (c) 44 (d) 55

264. If M is the mean of n observations $x_1 - k$, $x_2 - k$, $x_3 - k$,……… $x_n - k$ where k is any real number ,then what is the mean of $x_1$, $x_2$, $x_3$,….., $x_n$ ? **(2021-I)**

(a) $M$ (b) $M + k$ (c) $M - k$ (d) $kM$

265. What is the sum of deviations of the variate values 73, 85, 92, 105, 120 from their mean ? **(2021-I)**

(a) $-2$ (b) $-1$ (c) 0 (d)5

266. Let x be the HM and y be the GM of two positive numbers m and n If $5x = 4y$ , then which one of the following is correct ? **(2021-I)**

(a) $5m = 4n$ (b) $2m = n$ (c) $4m = 5n$ (d) $m = 4n$

267. If the mean of a frequency distribution is 100 and the coefficient of variation is 45%, then what is the value of the variance ? **(2021-I)**

(a) 2025 (b) 450 (c) 45 (d) 4.5

268. For which of the following sets of numbers do the mean ,median and mode have the same value ? **(2021-I)**

(a) 12, 12, 12, 12, 24 (b) 6, 18, 18 ,18 ,30 (c) 6, 6, 12, 30, 36 (d) 6 ,6, 6 12, 30

269. The mean of 12 observations is 75 . If two observations are discarded ,then the mean of the remaining observations is 65 .What is the mean of the discarded observations ?

(a) 250 (b) 125 (c) 120 (d) cannot be determined due to insufficient data

270. The average of a set of 15 observations is recorded ,but later it is found that for one observation ,the digit in the tens place was wrongly recorded as 8 instead of 3 .After correcting the observation , the average is

(a) reduced by $\frac{1}{3}$ ( b) increased by $\frac{10}{3}$ (c ) reduced by $\frac{10}{3}$ (d) reduced by 50

271. If A={2,3}, B={4,5}, C={5,6}, then what is the number of elements in A x (B ∩ C)?**(2010-II)**

(a) 2 (b) 4 (c) 6 (d) 8

**272. Let $N$ be the set of natural numbers and $A = \{n^2 | n \in N\}$ and $B = \{n^3 | n \in N\}$. Which of the following is correct?(2010-II)**

**(a)$A \cup B = N$ (b) $A \cap B$must be a finite set.**

**(c) The complement of $A \cup B$is an infinite set.**

**(d) $A \cap B$must be a proper subset of $\{m^6 | m \in N\}$.**

**273. Let $f: R \to R$ defined by $f(x) = \frac{|x|}{x}, x \neq 0, f(0) = 2$. what is the range of $f$?(2009-II)**

**(a){1,2} (b) {-1,1} (c) {-1,1,2} (d) {1}**

**274. Let $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \in R - \{0\}$, then $f(x)$ is equal to**

**(a) $x^2$ (b) $x^2 - 1$ (c) $x^2 - 2$, when $|x| \geq 2$ (d) none**

**275.If $A, B, C$ are three sets, then what is $A - (B - C)$ equal to?(2008-II)**

**(a)$A - (B \cap C)$ (b) $(A - B) \cup C$**

**(c) $(A - B) \cup (A \cap C)$ (d) (A-B)$\cup$ $(A - C)$**

276. If a set $A$ contains 4 elements, then what is the number of elements in $A \times p(A)$?**(2008-II)**

(a) 16 (b) 32 (c) 64 (d) 128

277. If $A \& B$ are two subset of a set $X$, then what is $A \cap (A \cup B)'$?**(2008-II)**

(a) $A$ (b) $B$ (c) $\emptyset$ (d) $A'$

278. If $X$ & $Y$ are any two non – empty sets then what is $(X - Y)'$ equals to?**(2009-I)**

(a) $X' - Y'$ (b) $X' \cap Y'$ (c) $X' \cup Y$ (c) $X - Y'$

279. If x is an integer and satisfy $9 < 4x - 1 \leq 19$, then x is an element of which one of the following sets

(a) {3, 4} (b) {2, 3, 4} (c) {3, 4, 5} (d) {2, 3, 4, 5}

280. The domain of the function $f = \frac{1}{\sqrt{9-x^2}}$ is

(a) $-3 \leq x \leq 3$ (b) $-3 < x < 3$ (c) $-9 \leq x \leq 9$ (d) $-9 < x < 9$

281. Expansion of $\sqrt{9 - x^2}$ is valid if

(a) $-3 < x < 3$ (b) $|x| > 3$ (c) $|x| \leq 3$ (d) $-1 < x < 1$

282. If *A, B, C* are three sets and Uis the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$then what is the value of $n(A' \cap B')$ ?

(a) 100 (b) 200 (c) 300 (d) 400

283.If $A \subseteq B$, then $B' - A'$ is equal to

(a) $A'$ (b) $B'$ (c) A–B (d) $\boldsymbol{\phi}$

284.The range of function $f(x) = |x|$ is

(a) $(0,\alpha)$ (b) $(-\alpha,0)$ (c) $[0,\alpha)$ (d) none

285.Let $R$ be a relation in $N$ defined by $R=\{(x,y)/x+2y=8\}$. The range of $R$ is

(a) $\{2,4,6\}$ (b) $\{1,2,3\}$ (c) $\{1,2,3,4,6\}$ (d) none

286. Two finite sets have $m$ and $n$ elements. The total number of subsets of the first set is 56 more than total number of subsets of second set. The values of $m$ and $n$ are **(2009-I)**

(a) 7,6 (b) 6,3 (c) 5,1 (d) 8,7

For the next three (03) items that follow:-**(2014-II)**

Consider the function $f(x)=\frac{x-1}{x+1}$

287. What is $\frac{f(x)+1}{f(x)-1}+x$ equal to?

(a) 0 (b) 1 (c) $2x$ (d) $4x$

288. What is $f(2x)$ equal to?

(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{f(x)+1}{3f(x)+1}$ (c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x)+3}{3f(x)+1}$

289. What is $f\left(f(x)\right)$ equal to?

(a) $x$ (b) $-x$ (c) $-\frac{1}{x}$ (d) none

290. Let $f=\{(1,1),(2,4),(0,-2),(-1,-5)\}$ be a linear function from $z$ to $z$. Then $f(x)$ is

(a) $3x-2$ (b) $6x-8$ (c) $5x-2$ (d) $7x+2$

291. The domain & range of the function $f(x) = 2-|x-5|$ is

(a) Domain $=R^{+}$, Range $=(-\infty, 1]$ (b) Domain $=R$, Range $=(-\infty, 2]$
(c) Domain $=R$, Range $=(-\infty, 2)$ (d) Domain $=R^{+}$, Range $=(-\infty, 2]$

292. The domain of the function $f$ defined as $f(x)=\sqrt{\frac{x+3}{(2-x)(x-5)}}$ is

(a) $(-\infty,-3]\cup(2,5)$ (b) $(-\infty,-3)\cup(2,5)$ (c) $(-\infty,-3]\cup[2,5]$ (d) none of these

293. What is the domain of the function $f(x)=3^{x}$? (2021)

(a) $(-\infty,\infty)$ (b) $(0,\infty)$ (c) $[0,\infty)$ (d) ) $(-\infty,\infty)-\{0\}$

294. Consider the following statements: **(2021-I)**

1.The null set is a subset of every set.

2. Every set is a subset of itself.

3. If a set has 10 elements ,then its power set will have 1024 elements

Which of the above statements is/are correct?

(a) 1 and 2 only (b) 2 and 3only (c) 1 and 3 only (d) 1,2 and 3

295. If $A=\{\{1,2,3\}\}$ *then* how many elements are there in the power set of $A$ ? **(2022-I)**

(a) 1 (b) 2 (c) 4 (d) 8

296.Consider all the subsets of the set $A=\{1,2,3,4\}$. How many of them are supersets of the set $\{4\}$ ? ? **(2022-I)**

(a) 6 (b) 7 (c) 8 (d) 9

297. Consider the following statements in respect of two non-empty sets A and B : **(2022-I)**

1. $x \notin (A \cup B) \Rightarrow x \notin A \text{ or } x \notin B$

2. $x \notin (A \cap B) \Rightarrow x \notin A \text{ and } x \notin B$

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) both 1 and2 (d) neither 1 nor 2.

298. Consider the following statements in respect of two non-empty sets A and B : **(2022-I)**

1. $A \cup B = A \cap B \ \ iff A = B$

2. $A \triangle B = \emptyset \ \ iff A = B$

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) both 1 and2 (d) neither 1 nor 2.

299. What is the domain of the function $f(x) = \sqrt{1-(x-1)^2}$ ?

(a) (0,1) (b) [-1,1] (c) (0,2) (d) [0,2]

300. Consider the following statements: **(2022-II)**

1. If $f$ is the subset of $Z \times Z$ defined by $f = \{(xy, x-y)/ \ \ x, y \ \in Z\}$ ,then $f$ is a function from $Z$ to $Z$ .

2. If $f$ is the subset of $N \times N$ defined by $f = \{(xy, x+y)/ \ \ x, y \ \in N\}$ ,then $f$ is a function from $N$ to $N$.

Which of the statements given above is / are correct ?

(a) 1 only (b) 2 only (c) both 1 and 2 (d) neither 1nor 2

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# CHAPTER-1 : RELATIONS AND FUNCTIONS

**Relation:** Let A and B are any two non-empty sets. Then any subset $R$ of $A \times B$( i.e $\subseteq A \times B$) is called a relation from A to B .

$(a, b) \in R$ is also written as $aRb$ and is read as $'a'$ is related to $'b'$ by the relation $R$ and $(a, b) \in R$ is also written as $aRb$ and is read as $'a'$ is related to $'b'$ by the relation $R$ $(c, d) \notin R$ is also written as $aRb$ and is read as $'a'$ is not related to $'b'$ by the relation $R$.

Example: Let $A = \{1,2,3\}$ and $B = \{a, b, c\}$ ,then

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$$

(i) $R_1=\{(1, a), (2, b), (2, c)\}$ is a relation from A to B as $R_1 \subseteq A \times B$
(ii) $R_2=\{(a, 1), (c, 2), (b, 3)\}$ is a relation from B to A as $R_2 \subseteq B \times A$
(iii) $R_3=\{(1, a), (2, b), (1,1)\}$ is not a relation from A to B as $(1,1) \notin A \times B$

**Types of Relations:**

1. **Empty relation (or void relation) :** A relation R on a set A is said to be empty relation ,if

$R = \emptyset \subseteq A \times A$

Example: Relation R on set $A = \{2,4,6,8\}$ defined by $R = \{(a, b)/a + b \text{ is odd}\}$ is an empty relation on A .

2. **Universal relation:** A relation R on a set A is said to be universal relation ,if

$R = A \times A \subseteq A \times A$

Example: Relation R on set $A = \{2,4,6,8\}$ defined by $R = \{(a, b)/a + b \text{ is even}\}$ is a universal relation on A .

3. **Identity relation:** A relation $I_A$ on a set A is said to be identity relation ,if

$I_A = \{only(a, a) \forall\, a \in A\}$

Example: Relation $I_A$ on set $A = \{2,4,6,8\}$ defined by $R = \{(a, b)/a - b = 0, \forall a, b \in A\}$ is an identity relation on A .

4. **Reflexive relation:** A relation R on a set A is said to be reflexive ,if $(a, a) \in R$ , $\forall\, a \in A$

Example 1: $R = \left\{(a, b)/a\,, b \in N \text{ and } \frac{a}{b} \in Z\right\}$ is a reflexive relation on $N$ ,as $(a, a) \in R\ \forall\, a \in N$,since $\frac{a}{a} = 1 \in Z$.

Example 2: Let $A = \{1,2,3\}$,then

$R_1=\{(1,1), (1,2), (1,3), (2,2)\}$ is not a reflexive relation on A as $(3,3) \notin R_1$

$R_2=\{(1,1), (1,2), (1,3), (2,2), (3,3), (3,1)\}$ is a reflexive relation on A .

$R_3=\{(1,1), (2,2), (3,3)\}$ is the smallest reflexive relation on A .

5. **Symmetric relation :** A relation R on a set A is said to be symmetric ,if for every

$(a, b) \in R \Rightarrow (b, a) \in R$

Example 1: $R = \left\{(a, b)/a\,, b \in N \text{ and } \frac{a}{b} \in Z\right\}$ is not symmetric relation on $N$ , as $(2,1) \in R$ ,since $\frac{2}{1} = 2 \in Z$ but $(1,2) \notin R$, since $\frac{1}{2} \notin Z$.

Example 2: Let $A = \{1,2,3\}$,then

$R_1=\{(1,1),(1,2),(2,1),(2,2)\}$ is a symmetric relation on A .

$R_2=\{(1,1),(2,2),(3,3)\}$ is a symmetric relation on A .

$R_3=\{(1,1)\} or \{(2,2)\} or \{(3,3)\}$ are the smallest symmetric relation on A .

$R_4=\{(1,3),(1,2),(2,1)\}$ is not a symmetric relation on A as $(3,1) \notin R_4$.

6. **Transitive relation:** A relation R on a set A is said to be transitive ,if for

$(a,b) \in R \text{ and } (b,c) \in R \Rightarrow (a,c) \in R \ \forall a,b,c \in A$

i.e $aRb$ and $bRc \Rightarrow aRc$

Example: $R = \left\{(a,b)/a,b \in N \text{ and } \frac{a}{b} \in Z\right\}$ is a transitive relation on $N$ .

Sol$^n$: ,Let $(a,b) \in R$ and $(b,c) \in R$ .

$\Rightarrow \frac{a}{b} \in Z$ and $\frac{b}{c} \in Z$

$\Rightarrow \frac{a}{b} \times \frac{b}{c} \in Z$

$\Rightarrow \frac{a}{c} \in Z$

$\Rightarrow (a,c) \in R$

$\therefore R$ is transitive relation on $N$ .

7. **Equivalence relation:** A relation R on a set A is said to be equivalence ,if it is simultaneously reflexive, symmetric and transitive.

Example: Relation $R$ on set $A = \{1,2\}$ defined by $R = \{(1,1),(2,2),(1,2),(2,1)\}$ is equivalence.

8. **Inverse relation:** If R is a relation from set A to set B ,then the inverse relation of R is denoted by $R^{-1}$ is a relation from B to A defined by $R^{-1} = \{(b,a)/(a,b) \in R, \text{where } a \in A \text{ and } b \in B\}$

Note: (i) $R^{-1} \subseteq B \times A$

(ii) $(b,a) \in R^{-1} \Rightarrow (a,b) \in R$

(iii) Domain of $R^{-1} = \text{Range of } R$

(iv) Range of $R^{-1} = \text{Domain of } R$

9. **Anti-symmetric relation:** A relation R on a set A is said to be anti-symmetric if $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b \quad \forall, a,b \in A$.

**Equivalence class:** Let R be an equivalence relation on set A ,then the equivalence class of $a \in A$ is denoted by $[a]$ and is defined as $[a] = \{x/x \in A \text{ and } xRa \text{ i.e } (x,a) \in R\}$

= subset of A containing all the elements related to $a$ through relation R.

Example: If A= $\{1,2,3,4,5\}$ and R is an equivalence relation on A defined by $R = \{(x,y)/x - y \text{ is even and } x,y \in A\}$ ,find $[1]$

Sol$^n$: $[1] = \{x \in A \,/\, xR1\} = \{x \in A/x - 1 = even\} = \{1,3,5\}$

**Properties of Equivalence relation:**

(i) If $R_1$ and $R_2$ are equivalence relations then $R_1 \cap R_2$ is also an equivalence relation.
(ii) If $R_1$ and $R_2$ are equivalence relations then $R_1 \cup R_2$ may not necessarily be an equivalence relation.
(iii) If $R$ is an equivalence relation ,then $R^{-1}$ is also an equivalence relation.

**Proof(i):** Reflexive: $(a,a) \in R_1$ *and* $(a,a) \in R_2$ as $R_1 and\, R_2$ are reflexive.

$\Rightarrow (a,a) \in R_1 \cap R_2 \;\; \forall a$

$\therefore$ $R_1 \cap R_2$ is reflexive.

*Symmetric*: Let $(a,b) \in R_1 \cap R_2$

$\Rightarrow$ $(a,b) \in R_1$ and $(a,b) \in R_2$

$\Rightarrow$ $(b,a) \in R_1$ and $(b,a) \in R_2$ $[\because R_1 \text{ and } R_2 \text{ symmetric}]$

$\Rightarrow (b,a) \in R_1 \cap R_2$

$\therefore$ $R_1 \cap R_2$ is symmetric .

*Transitive*: Let $(a,b) \in R_1 \cap R_2$ *and* $(b,c) \in R_1 \cap R_2$

$\Rightarrow$ $(a,b) \in R_1$ , $(b,c) \in R_1$ and $(a,b) \in R_2$ , $(b,c) \in R_2$

$\Rightarrow$ $(a,c) \in R_1$ and $\Rightarrow$ $(a,c) \in R_2$ $[\because R_1 \text{ and } R_2 \text{ transitive}]$

$\Rightarrow (a,c) \in R_1 \cap R_2$

$\therefore$ $R_1 \cap R_2$ is transitive.

*Hence* $R_1 \cap R_2$ is an equivalent relation.

**Proof(ii):** Let $A = \{1,2,3\}$ and two equivalent relations

$R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$R_2 = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$

Now $R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$ is not an equivalent relation as

$(1,2) \in R_1 \cup R_2, (2,3) \in R_1 \cup R_2\, but\; (1,3) \notin R_1 \cup R_2,$

*Hence* $R_1 \cup R_2$ is not an equivalent relation.

**Proof(iii):** Reflexive: $(a,a) \in R \;\; \forall a$ $[\because R \text{ is reflaxive}]$

$\Rightarrow (a,a) \in R^{-1} \;\; \forall a$ $[\because \text{by definition of } R^{-1}]$

$\therefore$ $R^{-1}$ is reflexive .

*Symmetric*: Let $(a,b) \in R^{-1}$

$\Rightarrow (b,a) \in R \quad [\because \textit{by definition of } R^{-1}]$

$\Rightarrow (a,b) \in R \quad [\because R \textit{ is symmetric}]$

$\Rightarrow (b,a) \in R^{-1} \quad [\because \textit{by definition of } R^{-1}]$

$\therefore$ $R^{-1}$ is symmetric.

$Transitive$: Let $(a,b) \in R^{-1}$ $and$ $(b,c) \in R^{-1}$

$\Rightarrow (b,a) \in R$ and $(c,b) \in R \quad [\because \textit{by definition of } R^{-1}]$

$\Rightarrow (c,b) \in R$ and $(b,a) \in R$

$\Rightarrow (c,a) \in R \quad [\because R \textit{ is transitive}]$

$\Rightarrow (a,c) \in R^{-1} \quad R \quad [\because \textit{by definition of } R^{-1}]$

$\therefore$ $R^{-1}$ is transitive.

$Hence$ $R^{-1}$ is an equivalent relation.

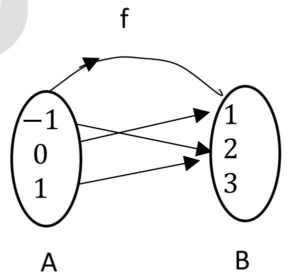
**Function**: Let A and B be two non-empty sets .Then the relation $f$ $(i.e\ f \subseteq A \times B)$ is called a function (or mapping) from A to B if for every $x \in A$ there exist exactly one (unique) $y \in B$, where $(x,y) \in f$

The function $f$ from $A$ to $B$ is denoted by $f: A \to B$ or A $\xrightarrow{f}$ B

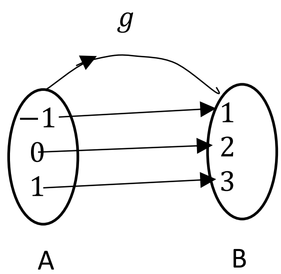
and $(x,y) \in f$ is written as $y = f(x)$ ,where $y$ is called image of $x$ under $f$ (or value of $f$ at $x$ ) and $x$ is called the pre image of $y$ under $f$ .

Example: Let the two sets be $A = \{-1,0,1\}$ and $B = \{1,2,3\}$ ,then

(i) The relation $f = \{(-1,2),(0,1),(1,2)\}$ is a function from A to B , defined by $f(x) = x^2 + 1$ (called quadratic function)

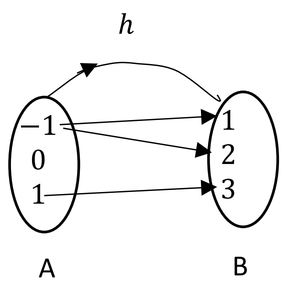



(ii) The relation $g = \{(-1,1),(0,2),(1,3)\}$ is a function from A to B , defined by $g(x) = x + 2$ (called linear function)



(iii)The relation $h = \{(-1,1), (-1,2), (1,3)\}$ is not a function from A to B

as $-1 \in A$ has two images 1,2 ,also $0 \in A$ has no image in B .




**Note:** If $f: A \to B$ then

(i) Domain of function $f = D_f = \text{set } A$

(ii) Range of function $f = R_f = \text{set of all } f \text{ images of all } x \in A$

$= \{y \in B / f(x) = y \quad \forall\, x \in A\} \subseteq B$

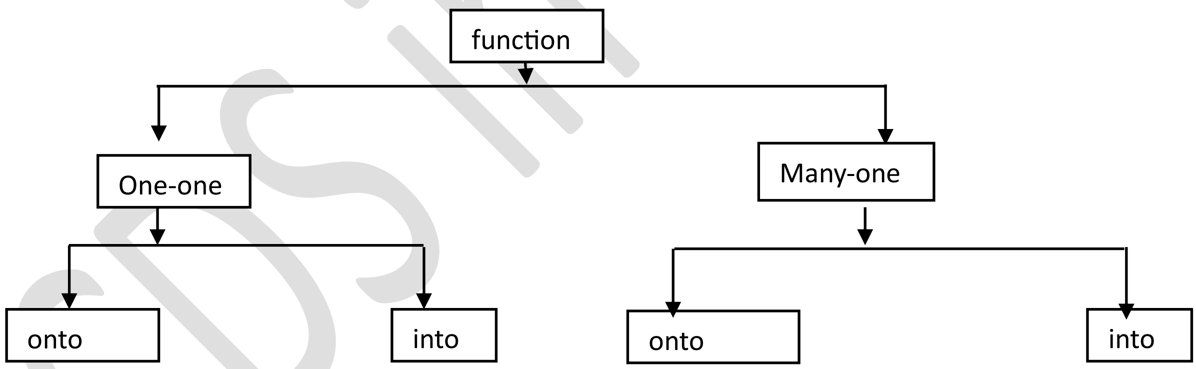
(iii) Co-domain of function $f = set\ B$

(iv) One or more elements of set A may have same $f$ image in set B .

(v) There may be some elements in set B which may not have any pre-image in set A .

(vi) Every function is a relation but every relation is not a function.

## Types of functions:




**One-one function( or injective function):** A function $f: X \to Y$ is said to be one-one if distinct elements in $X$ have distinct $f$ image in $Y$ .

i.e $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \quad \forall\ x_1, x_2 \in X$

or

$x_1 \neq x_2 \Rightarrow \quad f(x_1) \neq f(x_2)$ , $\forall\ x_1, x_2 \in X$

**Many – one function**: A function $f: X \to Y$ is said to be many-one if two or more elements in $X$ have the same $f$ image in $Y$ i.e if there exists $x_1 \neq x_2$ but $f(x_1) = f(x_2)$

**Onto function(or surjective function):** A function $f: X \to Y$ is said to be (one-one or many-one) onto if each elements of $Y$ (co-domain) is the image of at least one element of $X$ (domain) .

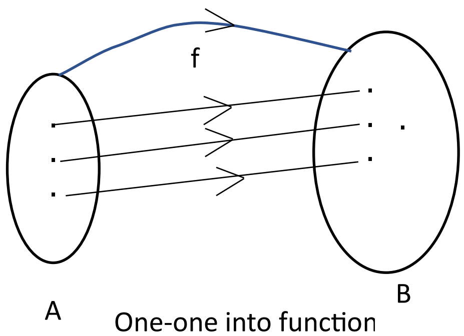
**Note**: For onto function range of $f$ =co-domain of $f$

**Into function**: A function $f: X \to Y$ is said to be (one-one or many-one) into if there exists at least one element in $Y$ which is not the f image of any element of $X$.

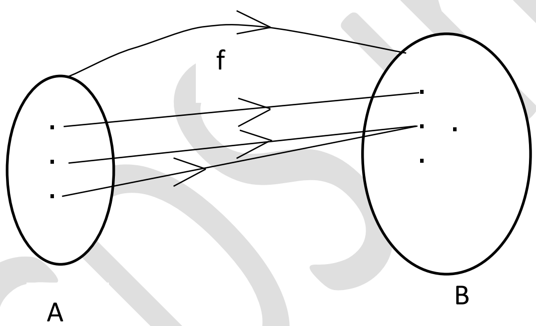
**Note**: For into function range of $f$ $\neq$ co-domain of $f$

One-one onto function(or bijective function): A function $f: X \to Y$ is said to be bijective ,if it is both one-one and onto.

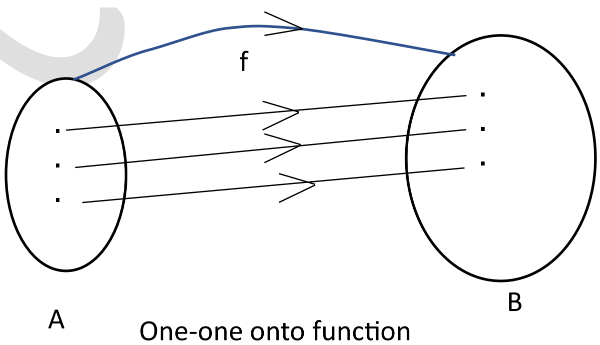
**Note**: For bijective function $n(X) = n(Y)$.



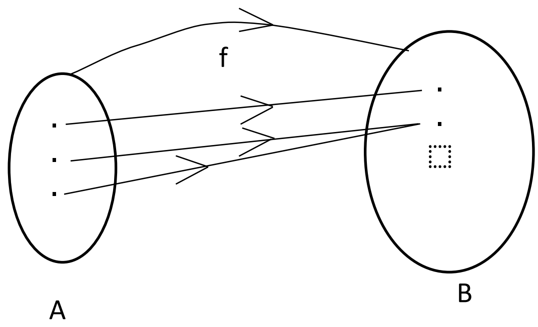

One-one into function




Many –one into function




One-one onto function




Many –one onto function

**Working rule for checking bijective function**:

**For one–one:** Take $f(x_1) = f(x_2)$ and then simplify it ,if result is $x_1 = x_2$ ,then function $f$ is one-one ,otherwise function $f$ is many-one.

**For onto:** Take $f(x) = y$ and then express $x$ in terms of $y$.If for every $y \in Y$(co-domain set ) that $x \in X$ (domain set) then function $f$ is onto, otherwise function $f$ is into.

**Identity function**: A function $f: X \to X$ is said to be identity function if $f(x) = x$ , $\forall\, x \in X$

**Inverse function**: If $f: X \to Y$ is bijective ,then the function $g: Y \to X$ is said to be inverse of function $f$,if if $f(x) = x \Rightarrow g(y) = x$ ,where $x \in X$ and $y \in Y$ and is denoted by $f^{-1}$.

**i.e** $f(x) = \mathbf{y} \Rightarrow \boldsymbol{f^{-1}(y) = x}$

**Working rule for finding inverse of function $\boldsymbol{f(x)}$ if exists:**

Step1: Put $y = \mathrm{f(x)}$.

Step2: Express $x$ in terms of $y$ .

Step3: Replace $x$ by $f^{-1}(y)$.

Step4: Replace $y$ by $x$ from both sides, which will be the required inverse of $\boldsymbol{f(x)}$

Example: Find $f^{-1}(x)$ where $f(x) = \frac{-5+5x}{-3-3x}$ $, x \neq -1$

Sol$^{n}$: $y = \frac{-5+5x}{-3-3x}$

$\Rightarrow -5 + 5\mathrm{x} = -3\mathrm{y} - 3\mathrm{xy}$

$\Rightarrow \mathrm{x}(5 + 3\mathrm{y}) = 5 - 3\mathrm{y}$

$$\Rightarrow \mathrm{x} = \frac{5 - 3y}{5 + 3y}$$

$$\Rightarrow f^{-1}(y) = \frac{5 - 3y}{5 + 3y}$$

$\Rightarrow f^{-1}(x) = \frac{5-3x}{5+3x}$.

**Equal functions(or identical functions):**Two functions $f: X \to Y$ and $g: X \to Y$ are said to be equal if

(i) Domain of $f(x)$ = Domain of $g(x)$
(ii) Range of $f(x)$ = Range of $g(x)$
(iii) $f(x) = g(x)\ \forall\ x \in D_f\ and\ x \in D_g$

Example: Let $A = \{-1,0,1,2\}$ and $B = \{-4,-2,0,2\}$ and $f, g: A \to B$ be functions defined by

$f(x) = x^2 - x$ and

$g(x) = 2\left|x - \frac{1}{2}\right| - 1$ , $x \in A$. Check if $f and\ g$ are equal .

Sol$^n$: Here Domain of $f(x)$ = Domain of $g(x) = set\ A$

$f(-1) = 2$ , $g(-1) = 2$

$f(0) = 0$ , $g(0) = 0$

$f(1) = 0$ , $g(1) = 0$

$f(2) = 2$ , $g(2) = 2$

Range of $f(x)$ = Range of $g(x) = \{0,2\}$

$f(x) = g(x)\ \forall\ x \in A$

$\therefore$ $f(x)$ and $g(x)$ are equal functions.`

**Composition of two functions (or function of function of product function):**

Let A, B and C be three non-empty sets and $f: A \to B$ and $g: B \to C$ be two functions .

Now $f: A \to B$ and so by definition of function for every $a \in A$ there is a unique image $f(a)$ in $B$.

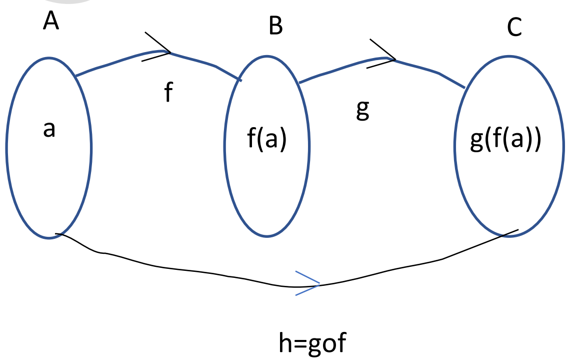
Again $g: B \to C$ and so by definition of function for every $f(a) \in B$ there is a unique image $g(f(a))$ in $C$.

*Thus* for every $a \in A$ there is a unique image $g(f(a))$ in $C$.

*This* function $h: A \to C$ is called composition of two functions $f$ and $g$ and is denoted by $gof$.

**Definition**: Let $f: A \to B$ and $g: B \to C$ be two functions . Then the function $gof: A \longrightarrow C$ is defined by $gof(x) = g(f(x))\ \forall\ x \in A$ is called composition of $f$ and $g$.

Similarly $fog(x) = f(g(x))$



**Note:** (i) $fof(x) = x \Rightarrow f^{-1} = f$

(ii) $gof(x) = x \Rightarrow f^{-1} = g$

**Properties of composite function:**

(i) $fog(x) \neq gof(x)$ i.e not commutative.

(ii) $(fog)o\,h(x) = f\,o(g\,o\,h)(x)$ i.e associative.

(iii) If $f(x)$ and $g(x)$ are one-one ,then $gof(x)$ is one-one.

(iv) If $f(x)$ and $g(x)$ are onto ,then $gof(x)$ is onto.

(v) If $f(x)$ and $g(x)$ are bijective ,then $gof(x)$ is bijective.

**Even function and odd function:** A function $f(x)$ is said to be an even function if $f(-x) = f(x)$ and a function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$.

Example: $f(x) = \cos x$ is an even function and $f(x) = \sin x$ is an odd function.

**Periodic function:** A function $f(x)$ is said to be periodic function of $\lambda$ if $f(x+\lambda) = f(x)\ \forall x \in R$

where $\lambda$ is the least positive number.

Example: $f(x) = \sin x$ is a periodic function of period $2\pi$ as

$$\sin(2\pi + x) = \sin(4\pi + x) = \cdots = \sin x.$$

**Properties of periodic function:** If $f(x)$ is periodic function of period $\lambda$ ,then

(i) $kf(x)$ is a periodic function of period $\lambda$.

(ii) $f(x+k)$ is a periodic function of period $\lambda$.

(iii) $f(x) \pm k$ is a periodic function of period $\lambda$

(iv) $cf(kx+d)$ is a periodic function of period $\frac{\lambda}{|k|}$

(v) $If\ f_1(x)$ and $f_2(x)$ are periodic of periods $\lambda_1\ and\ \lambda_2$ respectively ,then period of

$af_1(x) \pm b\, f_2(x) = LCM\ of\ \{\lambda_1, \lambda_2\}$ .

$(vi)$ Period of $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\text{cosec}\, x|$ is $\pi$.

$(vii)(\sin x, \cos x, \sec x, \text{cosec}\, x)^n$ has period $2\pi$ if $n$ is odd and period $\pi$ if $n$ is even.

$(viii)$ period of $|\sin x| + |\cos x| = \pi$

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# Practice Questions

## Section-I(I mark each)

**MCQ type**

1. Let $f : R - \left\{\frac{3}{5}\right\} \to R$ be defined by $(x) = \frac{3x+2}{5x-3}$ . Then

(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$
(c) $(fof)(x) = -x$ (d) $f^{-1}(x) = \frac{1}{19}f\ (x)$

2. Let $f: R \to R$ be defined by $f(x) = \frac{1}{x}$ , $\forall x \in R, then\ f\ is$ :

(a) $one - one$ (b) $onto$ (c) $bijective$ (d) $f\ is\ not\ defined$.

3. Consider the function $f: R \to \{0,1\}$ such that $f(x) = \begin{cases} 1 & if\ x\ is\ rational \\ 0 & if x\ is\ irrational \end{cases}$, which one of the following is correct?**(2010-II)**

(a) The function is one – one into (b) The function is many – one into

(c) The function is one – one onto (d) The function is many – one onto

4. Let R = {(1,3), (4,2), (2,4),(2,3), (3,1)} be a relation on the set A = {1,2,3,4}. Then R is **(AIEEE-2004)**

(a) a function (b) transitive (c) not symmetric (d) reflexive

5. Let R be a relation on the set N defined by $\{(x, y)| x, y \in N, 2x + y = 41\}$, then R is

(a) reflexive (b) symmetric (d) transitive (d) none

6.The relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ on a set $A = \{1,2,3\}$ is (2010-II)

(a)Reflexive, transitive but not symmetric.(b)Reflexive, symmetric but not transitive.

(c)Symmetric, transitive but not reflexive.(d)Reflexive but neither symmetric nor transitive

7. $f: R \to R$is a function defined by $f(x) = 10x - 7$. if $g = f^{-1}$, then $g(x)$ is equal to

(a) $\frac{1}{10x-7}$ (b) $\frac{1}{10x+7}$ (c) $\frac{x+7}{10}$ (d) $\frac{x-7}{10}$

8. If $f: R \to R,\ g: R \to R \& g(x) = x + 3$ and $(fog)(x) = (x + 3)^2$, then what is the value of $f(-3)$?**(2010-I)**

(a)-9 (b) 0 (c) 9 (d) 3

9. The function $f(x) = \frac{x}{x^2+1}$ from $R$ to $R$ is**(2010-I)**

(a) One-one and onto (b) Onto but not one-one

(c) Neither one-one nor onto (d)One-one but not onto

10. Let $g: R \to R$ be a function such that $g(x) = 2x + 5$. Then what is $g^{-1}(x)$?

(a) $\frac{x-5}{2}$ (b) $2x-5$ (c) $x-\frac{5}{2}$ (d) $\frac{x}{2}+\frac{5}{2}$

11. If $f(x)=e^x \& g(x)=\ln x$, then what is the value of $(gof)'(x)$?**(2009-II)**

(a) 0 (b) 1 (c) e (d) none

12. Which one of the following functions $f: R \to R$ is injective? **(2009-II)**

$f(x)=|x|,\ \forall x \epsilon R$ (b) $f(x)=x^2,\ \forall x \epsilon R$ (c) $f(x)=11,\ \forall x \epsilon R$ (d) $f(x)=-x,\ \forall x \epsilon R$

13. The number of onto function from the set {1, 2, 3, 4, …….., n} to itself is

(a) n! (b) n (c) $n^2$ (d) $n^n$

14. The function $f: R \to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is

(a) One – one but not onto (b) onto but not one – one

(c ) both one – one and onto (d) neither one – one nor onto

15. Let $Z$ be the set of integers and $f: Z \to Z$ be defined as $f(x)=x^2, x \in Z$,then the function is

(a) Bijective (c) Injective (c) Surjective (d) None of these

16. $A=\{p,q,r\}$ which of the following is an equivalence relation in $A$

(a) $R_1=\{(p,q),(q,r),(p,r),(p,p)\}$

(b) $R_2=\{(r,q),(r,p),(r,r),(q,q)\}$

(c) $R_3=\{(p,p),(q,q),(r,r),(p,q)\}$

(d) None of these

17. The inverse of the function $f(x)=x^3+5$ is

(a) $f^{-1}(x)=(x+5)^{\frac{1}{3}}$ (b) $f^{-1}(x)=(x-5)^{\frac{1}{3}}$ (c) $f^{-1}(x)=(5-x)^{\frac{1}{3}}$ (d) $f^{-1}(x)=5-x$

18. The domain of the function $f=\frac{1}{\sqrt{9-x^2}}$ is

(a) $-3 \le x \le 3$ (b) $-3 < x < 3$ (c) $-9 \le x \le 9$ (d) $-9 < x < 9$

19. Let $A=\{1,2,3,4\}$ and let $R=\{(2,2),(3,3),(4,4),(1,2)\}$ be a relation in A. Then R is

(a) reflexive (b)symmetric (c)transitive (d) none of these

20. Let $R=\{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A=\{3,6,9,12\}$. The relation is

(a) reflexive and transitive only (b) reflexive only

(c) an equivalence relation (d)reflexive and symmetric only

21. Let A={–1, 2, 5, 8}, B={0,1,3,6,7} and R be a relation 'is one less than' from A to B, then how many elements will R contain?**(2009-II)**

(a) 2 (b) 3 (c) 5 (d) 9

22. A relation R is defined on set Z of integers as follows. **(2006-I)**

$mRn \Leftrightarrow m+n$ is odd, then which of the following is/are correct?

(1) R is reflexive (2) R is symmetric (3) R is transitive

Select the correct answer using the code given below.

(a) 2 only (b) 2 & 3 (c) 1 & 2 (d) 1 & 3

23. If A= {1,2,3,4} and R={(1,1), (1,3), (2,2), (3,1), (3,4), (4,3), (4,4)} is a relation on A then which one of the following is correct?**(2008-II)**
(a) R is reflexive (b)R is symmetric and transitive
(c) R is transitive but not reflexive (d)R is neither reflexive nor transitive

24. If $f(x)=\begin{cases} 1, x \text{ is a rational number} \\ 0, x \text{ is an irrational number} \end{cases}$

What is/are the value(s) of $(fof)(\sqrt{3})$?**(2007-II)**
(a) 0 (b) 1 (c) both 0 & 1 (d) None of these

25. For any two real nos a & b, we define a R b if & only if $\sin^2 a+\cos^2 b=1$. The relation R is
(a) reflexive but not symmetric (b) symmetric but not transitive
(c) transitive but not reflexive (d) an equivalence relation

26. $R$ is a relation defined in $Z$ by $aRb$ if and only if $ab \geq 0$, then $R$ is
(a) reflexive (b) symmetric (c) transitive (d) equivalence

27. Let P={1,2,3} and a relation on set P is given by the set R={(1,2), (1,3), (2,1), (1,1), (2,2), (3,3) (2,3)} Then R is**(2012-II)**
(a) Reflexive, transitive but not symmetric(b) Symmetric, transitive but not reflexive
(c) Symmetric, reflexive but not transitive(d) None of the above

28. The function $f: N \rightarrow$ N, where N is the set of natural numbers, defined by $f(x) = 7x + 11$ is
(a) injective (b) surjective (c) bijective (d) None

29. Let N be the set of natural nos. A relation R on N defined as R={( $x, y$)/ $xy$>0 $x, y$ ∈N}. Which of the following is correct?**(2007-II)**
(a) R is symmetric but not transitive (b) R is reflexive but not symmetric
(c) R is symmetric & reflexive but not transitive (d) R is equivalence relation.

30. The function $f$: R→R defined by $f(x) = (x-1)(x-2)(x-3)$ is
(a) one-one but not unto (b) onto but not one-one
(b) both one-one & onto (d) neither one-one nor onto

31. Let N denote the set of all non-negative integers & Z denote the set of all integers. The function $f$:Z→N given by $f(x) = |x|$ is**(2014-I)**
(a) one-one but not onto (b) onto but non one-one
(c) both one-one & onto (d) neither one-one nor onto

32. Let $f: R \rightarrow R$ be defined by $f(x)$= $\frac{x-a}{x-b}$, where $a \neq b$, then $f$ is
(a) injective but not surjective (b) surjective but not injective
(c) bijective (d) none

33. The relation R in the set Z of integer given by R= $\{(a,b)/a-b$ is divisible by 5} is**(2013-II)**
(a)reflexive (b)reflexive but not symmetric
(c) symmetric& transitive (d) an equivalence relation

34. The function $f: R \to R$ defined by $f(x)$=($x^2$+1)$^{35}$ for all $x \in R$ is**(2008-II)**

(a) one-one but not onto (b) onto but not one-one
(c) neither one-one nor onto (d) both one-one and onto

35. Let $f:N \to Y$ be a function defined as $f(x) = 4x$+3 where $Y$={$y \in N/y = 4x$+3 for sine $x \in N$}. If $f$ is invertible then its inverse is **(AIEEE-2008)**
(a) $g(y)=\frac{y-3}{4}$ (b) $g(y)=\frac{3y-4}{3}$ (c) $g(y)=4+\frac{y+3}{4}$ (d) $g(y)=\frac{y+3}{4}$

36. If R & R' are symmetric relations (not disjoint) on a set A, then the relation R∩R' is

(a) reflexive (b) symmetric (c) transitive (d) none

37. If $f$: R×R→R such that $f(x+iy) = +\sqrt{x^2+y^2}$, then $f$ is
(a) many-one and into (b) one-one and onto
(c) many-one and onto (d) one-one and into

38. If $f(x) = ax + b$ & $g(x) = cx + d$such that $(g(x)) = g(f(x))$ , then which one of the following is correct?**(2014-II)**

(a) $f(c) = g(a)$ (b) $f(a) = g(c)$ (c)$f(c) = g(d)$ (d) $f(d) = g(b)$

39. Let a relation $R$ in the set $R$ of real numbers be defined as $(a, b) \in R$ if and only if $1 + ab > 0$ for all $a, b \in R$. The relation $R$ is
(a) reflexive & symmetric (b) symmetric & transitive
(c) only transitive (d) an equivalence relation

40 .If two functions $f$ & $g$ are defined as $f : R \to R, f(x) = \sin x$, $g: R \to R, g(x) = x^2$ then $fog$ $is$

(a) $\sin x + x^2$ (b) $(\sin x)^2$ (c) $\sin x^2$ (d) $\frac{\sin x}{x^2}$

For the next two (02) items that follow: **(2016-I)**
Let $f$ (x) be the greatest integer function and g(x) be the modulus function.

41. What is (gof) ($-\frac{5}{3}$) – (fog) ($-\frac{5}{3}$) equal to?

(a) −1 (b) 0 (c) 1 (d) 2

42. What is (fof) ( $-\frac{9}{5}$ ) + (gog) (− 2 ) equal to?

(a) −1 (b) 0 (c) 1 (d) 2

43. If $\int f(x)dx = f(x)$, then $\int \{f(x)\}^2 dx$ is equal to

(a) $\frac{1}{2}\{f(x)\}^2 + c$ (b) $\{f(x)\}^3 + c$ (c) $\frac{\{f(x)\}^3}{3} + c$ (d) $\{f(x)\}^2 + c$

44. Let g(x) = 1 + $x$ – $[x]$ when $[x]$ is greatest integer function. If f($x$) = $\begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ then f(g(x)) is equal to

(a) $x$ (b) 1 (c) f($x$) (d) g($x$)

45. Let $f: R \to R$ $such\ that\ f(x) = \frac{1}{1+x^2}, x \in R\ then\ f\ is$

(a) Injective (b) surjective (c) bijective (d) None

46. Let function $f: R \to R$ be defined by $f(x) = 2x + sinx\ \forall\ x \in R$. then $f$ is

(a) one-one & onto (b) One-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

47. If$f: R \to R\ defined\ by\ f(x) = x^2 + 1\ then\ the\ value\ of f^{-1}(17)\ and f^{-1}(-3)$are respectively

(a) $\emptyset, \{4, -4\}$ (b) $\{3, -3\}, \emptyset$ (c) $\emptyset, \{5, -5\}$ (d) $\emptyset, \{2, -2\}$

**Assertion –Reason Based Questions**

In the following questions ,a statement of Assertion (A) is followed by a statement of Reason (R) Choose the correct answer out of the following choices

(a) Both A and R true and R is the correct explanation of A .

(b) Both A and R true and R is not the correct explanation of A .

(c ) A is true but R is false .

(d)A is false but R is true .

1.Assertion(A) If $R$ is the relation defined in set $\{1,2,3,4,5,6\}$ as
$R = \{(a, b): b = a + 1\}$,then $R$ is reflexive.
Reason(R) The relation R in the set $\{1,2,3\}$ given by $R = \{(1,2), (2,1)\}$ is symmetric.
Ans (d)

Sol: $R = \{(a, b): b = a + 1\}$
$Here\ (1,1) \notin R\ as\ 1 \neq 1 + 1$
So is not reflexive.

Again in $R = \{(1,2), (2,1)\}, (1,2) \in R \Rightarrow (2,1) \in R$

So R is symmetric.

2. Assertion(A) The function $f: R \longrightarrow R$ given by $f(x) = x^3$ is injective.

Reason(R) The function $f: X \longrightarrow Y$ is injective if $f(x) = f(y) \Rightarrow x = y, \forall\ x, y \in X$

Ans: (a)

3. Assertion(A) The function $f: R^* \longrightarrow R^*$ given by $f(x) = \frac{1}{x}$ is one- one and onto.

Where $R^*$ is the set of all non zero real numbers .

Reason(R) The function $g: N \longrightarrow R^*$ given by $f(x) = \frac{1}{x}$ is one- one and onto.

Ans: (c)

4. Assertion(A)Let A and B be sets ,then the function$f: A \times B \longrightarrow B \times A$ such that $f(a,b) = (b,a)$ is bijective.

Reason(R) A function f is said to be bijective if it is both one-one and onto.

Ans: (a)

5.If R is a relation in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b): |a-b| is\ even\}$

Assertion(A) R is equivalence relation.

Reason(R) All elements of $\{1,2,3\}$ *are related to all elements of* $\{2,4\}$

Ans: (c)

6. Assertion(A): The modulus function $f: R \longrightarrow R$ given by $f(x) = |x|$ is neither one-one nor onto.

Reason(R):The signum function $f: R \longrightarrow R$ given by $f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$

is bijective.

Ans: (c)

7. Assertion(A): Let $A = \{2,4,6\}$ and $B = \{3,5,7,9\}$ and defined a function $f = \{(2,3),(4,5),(6,7)\}$

from $A$ to $B$,then f is not onto.

Reason(R): A function $f: A \longrightarrow B$ is said to be onto ,if every element of $B$ is the image of some elements of $A$ under $f$.

Ans: (b)

8. Assertion(A): Function $f: R \longrightarrow R$ given by $f(x) = [x] + x$ is one-one onto.

Reason(R): A function is said to be one-one onto ,if each element has unique image and range of $f(x)$ is equal to codomain of $f(x)$.

Ans: (a)

Sol: $[x]$ gives integral value and $[x]+x$ gives real value and $f(x)$ has no repeated value for any value of x ,so f is 1-1 onto

9. Assertion(A): Let a relation R defined from set A to A such that $A=\{1,2,3,4\}$ and

$R=\{(1,1),(2,2),(3,3),(1,3),(3,1)\}$,then R is transitive.

Reason(R): A relation R in set A is said to be transitive ,

if $(a,b)\in R$ and $(b,c)\in R \Rightarrow (a,c)\in R \quad \forall\, a,b,c\in A$

Ans: (a)

## Section-II(2 mark each)

1.Let R be the relation in the set Z of integers given by R = {(a, b): 2 divides a – b}.Show that the relation R transitive? Write the equivalence class [0]

2.Show that the relation R on defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.

3. Let A = R – {2} and B = R – {1}. If f : A → B is a function defined by$f(x)=\frac{x-1}{x-2}$, then show that f is one-one and onto

4. Check whether the function f : N → N defined by f(x) = 4 – 3x is one-one or not.

5. Prove that the function f(x) = $\sqrt{3}$ sin 2x – cos 2x + 4 is one-one in the interval $\left[\frac{-\pi}{6},\frac{\pi}{3}\right]$.

### Section-III(4 mark each)

1.Show that the function $f: R \to \{x\in R/-1<x<1\}$defined by $f(x)=\frac{x}{1+|x|}, x\in R$is one-one and onto function.

2.Let$A=\{\,1,2,3,4,\ldots..9\}$ and $R$ be a relation in $A\times A$ defined by $(\,a,b)R\,(c,d)if$ $a+d\ =\ b+c\ for\ (a,b),(c,d) in\ A\ \times A$. Prove that R is an equivalence relation and also obtain the equivalent class [(2,5)].

3. Prove that the function $f:[\,0,\infty)\to R\ given\ by\ f\ (x)=9x^2+\ 6x-5$ is not invertible. Modify the co-domain of the function $f$ to make it invertible, and hence find $f^{-1}$.

4. Check whether the relation R in the set R of real numbers, defined by

$R=\{(a,b):1+ab\ >0\}, is\ reflexive$,symmetric or transitive.

5. Let Y=$\{n^2/n\in N\}\subset N$. Consider $f: N\to Y$ as $f(n)=n^2$. Show that $f$

is invertible . Find the inverse of $f$.

6. Prove that the function *f:N→* $N$ *defined by*

$f(x)=x^2+x+1$ is one-one but not onto.

7. Let $f: N\to N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases} \quad \text{for all } n \in N.$$

Find whether the function $f$ is bijective.

8.Show that the function f : (– ∞, 0) → (–1, 0) defined by f(x)= $\frac{x}{1+|x|}$, x ∈ (– ∞, 0) is one-one and onto.

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**SAINIK SCHOOL IMPHAL**
**SUMMER BREAK ASIGNMENT 2025-26**
**SUB: PHYSICS**
**CLASS: 12**

## CHAP-1

## BASIC PROPERTIES OF CHARGE

1. Name any two basic properties of electric charge.

2. A body is charged $-2$C by rubbing it, what will happen to its weight?

3. The charge on a proton is $+1.6 \times 10^{-19}$ C and that on an electron is $-1.6 \times 10^{-19}$ C. Does it mean that the electrons have a charge $3.2 \times 10^{-19}$ C less than the charge of a proton? Justify your answer.

4. Is it possible for particle to carry a charge of $+2.4 \times 10^{-19}$ C? Justify your answer.

Q. (i) Example 1.1(5*), and (ii) Example 1.2,
(iii) Exercise 1.4 (7*), (iv) Exercise 1.5 (7*) and (v) Exercise 1.11(5*)

## COULOMB'S FORCE

5. Two point charges $q_1$ and $q_2$ are placed close to each other what is the nature of force between them when (i) $q_1q_2 < 0$ (ii) $q_1q_2 > 0$

6. How does the Coulomb force between two point charges depend upon
(i) the charge (magnitude and nature) of the two point charges?
(ii) the separation between the two point charges?
(iii) the dielectric constant of the intervening medium?

7. Force of attraction between to point charges placed at distance '$d$' apart in a medium is 'F'. What should be the distance apart in the same medium so that the force of attraction between them becomes 9 F?

8. A charge Q is to be divided on two objects. What should be the values of the charges on the objects so that the force between the objects can be maximum?

9. A charge q is placed at the centre of the line joining the two equal charges Q. Show that the system of three charges will be in equilibrium, if $q = -Q/4$.

10. Two particles A and B having charges $8.0 \times 10^{-6}$ C and $-2.0 \times 10^{-6}$ C respectively are held fixed with a separation of 20 cm. Where a third charged particle should be placed so that it does not experience a net electric force?

11. Three equal charges, each having a magnitude of $2.0 \times 10^{-6}$ C, are placed at the three corners of a right-angled triangle of side 3 cm, 4 cm and 5 cm. Find the force on the charge at the right-angled corner.

12. A system of charges q and 2q are separated by a distance of 1 m apart. Find the magnitude and position of the third charge so that this system is in equilibrium.

13. Two charged particles having charge $2.0 \times 10^{-8}$ C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.

14. Plot a graph showing the variation of coulomb force (F) versus

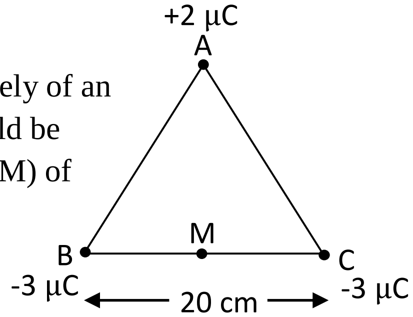
(i) $\left(\frac{1}{r^2}\right)$ where r is the distance between the two charges of each pair of charges: (1 μC, 2 μC) and (2 μC, – 3 μC).

(ii) dielectric constant, K, of the medium between the charges.

Interpret the graphs obtained.

15. How does the force on a charge due to another charge depend on the charges present nearby?

16. Two similar and equally charged identical metal spheres A and B repel each other with a force of $2 \times 10^{-5}$ N. A third identical uncharged sphere C is touched with A and then placed at the midpoint between A and B. Calculate the net electric force on C.




17. There point charges of are kept at the vertices, A, B and C respectively of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge to be placed at the mid-point (M) of side BC so that the charge at A remains in equilibrium?

18. Explain why $1\boldsymbol{C}$ defined as $\boldsymbol{1C = 1\,A\,s}$, is too big a unit for electric effects.

19. "Superposition principle should not be regarded as 'obvious', or equated with the law of addition of vectors." Explain the above statement.

20. Two particles, each having a mass of 5 g and charge $1.0 \times 10^{-7}$C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient.

21. Two free point charges + q and + 4q are a distance L apart. A third charge is placed so that the entire system is in equilibrium.

(a) Find the sign, magnitude and location of the third charge.

(b) Show that the equilibrium is unstable.

22. Two charged particles are placed at a distance of 1.0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge?

23. Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with force of 0.1 N, how many electrons were transferred from one sphere to the other during rubbing? Two charged particles having charge $2.0 \times 10^{-8}$ C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.

24. Two small spheres, each having a mass of 20 g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at equilibrium is found to be 4 cm. Find the charge on each sphere.

25. Two particles A and B having charges q and 2q respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped?

Q. (i) Exercise 1.2; (ii) Exercise 1.4(7*); (iii) Exercise 1.5(7*); (iv) Exercise 1.6; (v) Exercise 1.11; (vi) Exercise 1.12; (vii) Exercise 1.13(7*)

Q (i) Example 1.1, (ii) Example 1.3, (iii)Example 1.4, (iv) Example 1.5(7*), and (v) Example 1.6(10*)

## ELECTRIC FIELD

26. Define the physical quantity which has $NC^{-1}(Vm^{-1})$ as its unit. Is it a scalar or vector quantity? Justify your answer.

27. The electric force experienced by a charge of $1.0 \times 10^{-6}$ C is $1.5 \times 10^{-3}$N. Find the magnitude of the electric field at the position of the charge.

28. The electric field E due to a point charge at any point near it is defined as $E = \lim_{q_0 \to 0} \frac{F}{q_0}$ where $q_0$ is the test charge and F is the force acting on it. What is the physical significance of limit in this expression? Draw the electric filed lines of a point charge Q when (i) Q > 0 and (ii) Q < 0 .

29. A positively charged ball hags from a long silk thread. We want to measure E at a point in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge $q_0$ at the point and measure $F/q_0$. Will $F/q_0$ be less than, equal to or greater than E at that point in the question? Justify your answer.

30. In an electric field an electron is kept freely. If the electron is replaced by a proton, what will be the relation between the forces experienced by them?

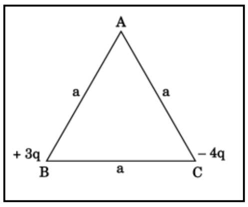
31. Two point charges of unknown magnitude and sign are a distance of *d* apart. The electric field is zero at one point between them. What can you conclude about the charges?

32. Two point charges of unknown magnitude and sign are a distance of d apart.
(a) If it is possible to have $E = 0$ at any point not between the charges but on the line joining them, what are the necessary conditions and where is the point located?
(b) Is it possible, for any arrangement of two point charges, to find two points (neither at infinity) at which $E = 0$? If so, under what conditions?

33. Two point charges of $+5\times10^{-19}$ C and $+20\times10^{-19}$ C are separated by a distance of 2 m. Find the point(s) on the line joining them at which electric field intensity is zero.

34. A point charge is brought in an electric field. How would the electric field at a nearby point be affected when point charge is + ve and − ve ?

35. Two point charges + 3q and – 4q are placed at the vertices ‘B’ and ‘C’ of an equilateral triangle ABC of side ‘a’ as given in the figure. Obtain the expression for (i) the magnitude and (ii) the direction of the resultant electric field at the vertex A due to these two charges.




36. A vertical electric field of magnitude $4.00 \times 10^5$ $NC^{-1}$ just prevents a water droplet of mass $1.00 \times 10^{-4}$ kg from falling. Find the charge on the droplet.

37. A charge particle of mass 1.0 g is suspended through a silk thread of length 40 cm in a horizontal electric field of $4.0 \times 10^4$ $NC^{-1}$. If the particles stays at a distance of 24 cm from the wall in equilibrium, find the charge on the particle.

38. A point charge q of mass m is released from rest in a uniform electric field.
(a) What is the nature of motion of the point charge?

(b) If the point charge were to enter this region perpendicular to the uniform electric field, what will be the nature of the path followed?

39. A ball of mass 100 g and having a charge of $4.9 \times 10^{-5}$ C is released from rest in a region where a horizontal electric field of $2.0 \times 10^{4}$ $NC^{-1}$ exists.
(a) Find the resultant force acting on the ball.
(b) What will be the path of the ball?
(c) Where will the ball be at the end of 2 s?

40. A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

41. A particle of mass *m* and charge *q* is thrown at a speed *u* against a uniform electric field E. How much distance will it travel before coming to momentary rest?

Q. (i) Example 1.7(10*); (ii) Example 1.8 (7*)

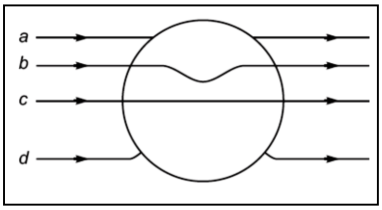
Q. Exercise 1.8 (5*)

## ELECTRIC FIELD LINE

42. What are electric lines of force? Give its important properties.

43. Diagrams for electric filed lines for system of charges:
(i) a +ve/–ve point charge point charge (ii) an electric dipole
(iii) a system of two unequal –ve/+ve charges separated by certain distance.

44. A metallic sphere is placed in a uniform electric field as shown in the figure.
Which path is followed by electric field lines and why?

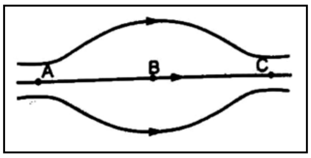



45. A point charge +Q is placed in the vicinity of a conducting surface. Draw the electric field lines between the surface and the charge.

46. Why the charge particle moved along the electric field line if the charged particle starts moving from rest; and not always if the charged particle was in motion?

47. Three point charges $+Q, -Q$ and + q are arranged at the vertex A, B and C respectively of an equilateral triangle of side $l$. Sketch qualitatively the field lines associated with this system of charges.

48. A, B and C are three point in an electric field region represented by the electric field lines as shown in the figure. Compare the strength of electric field at these points.

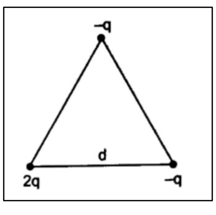



Q. (i) Exercise 1.7(7*); and (ii) Exercise 1.26(old text book)

## ELECTRIC DIPOLE

49. Two charges $q_1$ and $q_2$ separated by a small distance, satisfy the equation $q_1 + q_2 = 0$.
(i) What does it tell us about the charges?
(ii) Draw the electric field lines associated this system of charges.

50. Define electric dipole moment. Write its S.I. unit. Is it a scalar or a vector quantity?



51. Three charges are arranged on the vertices of an equilateral triangle as shown in figure. Find the dipole moment of the combination.

52. Derive the expression for the electric field of a dipole at a point on the (i) equatorial plane, and (ii) line [or axial] of the dipole. (iii) Draw a graph of E versus r for $r >> a$.

53. Show mathematically that the electric field intensity due to a short dipole at a distance '$d$' along its axis is twice the intensity at the same distance along the equatorial axis.

54. "The electric field due to a charge configuration with total charge zero is not zero; but for distances large compared to the size of the configuration, its field falls off faster than $\frac{1}{r^2}$, typical of field due to a single charge." Justify the above statement giving a suitable example.

Q. Example 1.9(7*) and Exercise 1.9(7*)

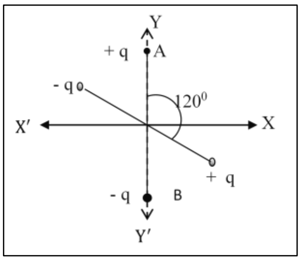
## DIPOLE PLACED IN A UNIFORM ELECRTIC FIELD

55. An electric dipole is held in a uniform electric field.
(i) Using suitable diagram, show that it does not undergo any translatory motion. What will happen if the field is non-uniform?
(ii) Derive an expression for the torque acting on it and specify its direction. And hence find the torque acting on the dipole in its stable equilibrium position and unstable equilibrium; also show diagrammatically how the dipole is oriented with respect to the field in the above positions.
(iii) Find the orientation of the dipole relative to the electric field for which torque on it is (i) maximum, and (ii) half of maximum.

56. What happens to an electric dipole when it is placed in a non-uniform electric field?

57. (i)When an electric dipole of dipole moment 'p' is placed in a uniform electric field E making a small angle $\theta$ with the field is set free, neglecting gravitational force on the dipole, show that the dipole executes SHM and hence determine the time period of oscillation.
(ii) If the dissipative force of atmospheric friction is considered, how would the dipole rest after the oscillation?

58. Two small identical electrical dipoles AB and CD, each of dipole moment 'p' are kept at an angle of $120^0$ as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field ($\vec{E}$) directed along + X direction, what will be the magnitude and direction of the torque acting on this?




59. An electric dipole of length 4 cm, when placed with its axis making an angle of $60^0$ with respect to a uniform electric field experiences a torque of $4\sqrt{3}$ Nm, calculate the magnitude of the electric field, if the dipole has charge of $\pm$ 9 nC.

Q. Exercise 1.10

## ELECTRIC FIELD AND GAUSS' THEOREM:

60. What is electric flux? Write its S. I. Units.

61. A circular ring of radius r made of a non-conducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through $180^0$. Does the flux of electric field change? If yes, does it decrease or increase?

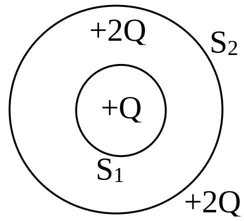
62. A small plane area is rotated in an electric field. In which orientation of the area is the flux of electric field through the area maximum? In which orientation is it zero?

63. A large plane charge sheet having surface charge density $\sigma = 2.0 \times 10^{-6}$ $Cm^{-2}$ lies in the X-Y plane. Find the flux of the electric field through a circular area of radius 1 cm lying completely in the region where x, y, z are all positive and with its normal making an angle of $60^0$ with the Z-axis.

64. State Gauss' theorem.
(i) Using Gauss's theorem, deduce an expression for the electric field at a point due to a uniformly charged infinite plane sheet. And show that the electric field at any point due to the uniformly charged infinite plane sheet is independent of the distance from it.
(ii) Use this law to derive an expression for the electric field due to an infinitely long straight wire of linear charge density $\lambda$ $Cm^{-1}$.
Draw a graph to show the variation of E with perpendicular distance 'r' from the line of charge.
(iii) Use this law to derive an expression for the electric field between two equal and unlikely charged sheets.

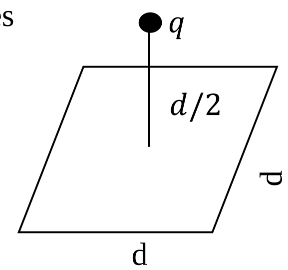
65. Using Gauss's law obtain the expression for the electric field due to a uniform charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for $r > R$ and $r < R$.
(i) Show mathematically that for the electric field filed any point outside the shell is same as, as if the entire charge of the shell is concentrated at the centre.
(ii) Why do you expect the electric field inside the shell to be zero according to this theorem?

66. $S_1$ and $S_2$ are two hollow concentric spheres enclosing charges Q and 2Q respectively as shown in the fig.
(i) What is the ratio of electric flux through $S_1$ and $S_2$?
(ii) How will the electric flux through the sphere $S_1$ change, if a medium of dielectric constant 5 is introduced in the space inside $S_1$ in placed of air?




67. A spherical rubber balloon carries a charge that is uniformity distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason.

68. (a) Define electric flux. Is it a scalar or a vector quantity? A point charges q is at a distance of d/2 directly above the centre of a square of side d, as shown in the figure.
Use Gauss' law to obtain the expression for the electric flux through the square.
(b) If the point charge is now moved to a distance 'd' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.




69. A charge 'q' is placed at the centre of a cube of side $l$. What is the electric flux passing through two opposite faces (each side) of the cube?

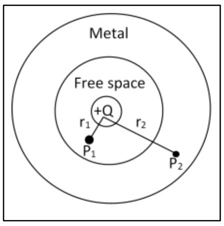
70. Two large parallel plane sheets have uniform charge densities $+\sigma$ and $-\sigma$. Determine the electric field (i) between the sheets, and (ii) outside the sheets.

71. Show that electric field across a charged spherical shell, of uniform surface charge density, is discontinuous.

72. "A cavity inside a conductor is shielded from outside electrical influences. It is worth noting that electrostatic shielding does not work the other way round". Justify the above statement.

73. Show that electric field inside a charged conductor is zero. And hence derive an expression for electric field on the surface of a charged conductor.

74. A small metal sphere carrying charge +Q is located at the centre of a spherical cavity in a large uncharged metal sphere as shown in the fig.
Use Gauss' theorem to find electric field at points $P_1$ and $P_2$.

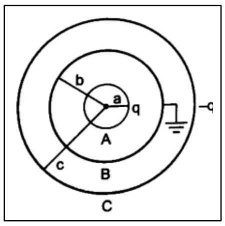



75. A charge Q is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell? If a point charge is brought close to the shell, will the field at the centre change? Does your answer depend on whether the shell is conducting or non-conducting?

76. A positive point charge (+$q$) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate.
Derive the expression for the electric field at the surface of a charged conductor. 3(09AI)

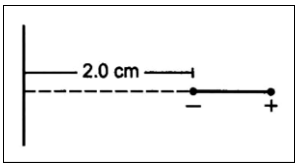
77. A spherical conducting shell of inner radius $r_1$ and outer radius $r_2$ has a charge '$Q$'. A charge '$q$' is placed at the centre of the shell.
($a$) What is the surface charge density on the ($i$) inner surface, ($ii$) outer surface of the shell?
($b$) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell.

78. As shown in the figure, three concentric thin spherical shells A, B and C of radii a, b, and c respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.




79. A charge of $4 \times 10^{-8}$ C is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of radius 5 cm.
(a) Find the electric field at a point 2 cm away from the centre.
(b) A charge of $6 \times 10^{-8}$ C is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.

80. An electric dipole consists of charges $\pm 2.0 \times 10^{-8}$ C separated by a distance of $2.0 \times 10^{-3}$ m. It is near a long line charge of linear charge density $4.0 \times 10^{-4}$ Cm$^{-1}$ as shown in the figure, such that the negative charge is at a distance of 2.0 cm from the line charge. Find the force acting on the dipole.




Q. (i) Example 1.10(10*) and (ii) Example 1.11(10*); (iii) Exercise 1.14(7*) (iv) Exercise 1.15
(v) Exercise 1.16(5*) (vi) Exercise 1.17(10*) (vii) Exercise 1.18 (viii) Exercise1.19(5*)
(ix) Exercise 1.20 (x) Exercise 1.20 (xi) Exercise 1.21(7*) (xii) Exercise 1.22
(xiii) Exercise 1.23(5*) and (xiv) Exercise 1.24(old)

# BIOLOGY CLASS - XII

# SUMMER VACATION HOME ASSIGNMENT 2025 – 2026

**CHAPTER 2**

1) How are the cells in an embryo sac organised?
2) Describe the structure of a typical angiosperm ovule with diagram.
3) "Incompatibility is the natural barrier in the fusion of gamete". Justify this statement.
4) What exactly is triple fusion? Where does it happen?
5) Explain the stages of development of a microspore into a pollen grain.
6) Describe the structure of a microsporangium with a neatly labelled diagram.
7) Differentiate between Geitonogamy & Autogamy.
8) Describe the composition of a pollen grain with labelled diagram.
9) Follow the growth of microsporocytes into mature pollen grains.
10) If an angiospermic plant has a diploid number of chromosomes of 16. Mention how many chromosomes are in the endosperm and antipodal cell.
11) At the end of microsporogenesis and megasporogenesis, what structures are formed?
12) Distinguish microsporogenesis from megasporogenesis. What type of cell division takes place during these events? Name the structure that results from these two events.
13) Describe the post-fertilization changes taking place in a flowering plant?

**CHAPTER 3**

1) Give a reason for the following statements :
   a. The first half of the menstrual cycle is called the follicular phase as well as the proliferative phase.
   b. The second half of the menstrual cycle is called the luteal phase as well as the secretory phase.
2) State the reason why the male testes are located outside the abdominal cavity.
3) State the male accessory glands and their function.
4) State the number of chromosomes in the following cells. Primary oocyte, secondary oocyte, ootid, and follicle.
5) Explain the function of the epididymis in male fertility.
6) How is polyspermy checked by the zona pellucida of the ovum?
7) A sperm has just fertilised a human egg in the fallopian tube. Write down the events that the fertilised eggs will go through till the implantation of the blastocyst in the uterus.
8) Explain the function of the epididymis in male fertility.
9) What do the term parturition and lactation mean?
10) What is meant by L.H. Surge? Write the role of L.H. State the levels of FSH, LH, and Progesterone simply by mentioning high or low around the 13th and 14th day and 21st to 23rd day.
11) Draw a diagram of the T.S. of seminiferous tubule of testis of an adult human male & label any four parts in it.
12) 'A fertilised egg is the blueprint of future development'. Explain the statement.
13) Differentiate between spermatogenesis and oogenesis.
14) Describe the hormonal control of the human male reproductive system with the help of a flow chart & highlight the inhibitory & stimulatory directions in it?
15) Explain the organisation of the mammary gland.

## ***XII Chemistry Summer Vacation Home Work***

*Unit I: Solutions*

1.What is the effect of temperature change on Henry's constant?

2. State Raoult's law. Raoult's law is a special case of Henry's law. Comment.

3. Define vapour pressure of a liquid. Discuss about the effect of dissolution of a non – volatile solute to vapour pressure of a liquid.

4. What are ideal and non – ideal solutions? Discuss various factors affecting involving in deviation from Raoult's law with suitable examples.

5. What is colligative property of a solution? How measurement of colligative property of a solution helps in the determination of molar mass of an unknown solute? (Derive the required relations for all the four colligative properties to calculate molar mass of the solute.)

6. What do you mean by abnormal molecular mass? What are factors causing abnormal molecular masses of the solute?

7. Define van't – Hoff factor. How it is related to degree of dissociation and degree of association, write the required relation and how it helps in the correction of abnormal molecular mass of solutes?

*Unit II: Electrochemistry.*

1. Define conductance of a conductor. How conductance of (i) electronic conductor (ii) Ionic conductor and (iii) semiconductors are affected by change in temperature and why so?

2.How specific conductance or conductivity of an electrolytic solution affected on dilution? Explain why.

3. Explain variation of conductances of weak and strong electrolytes on dilution.

4. Define electrode potential and standard electrode potential. How electrode potential of an electrode is determined by coupling with reference electrode like SHE? Explain.

5. What do you mean by electrochemical cells? What are its types?

6. Explain the working of the following electrochemical cells by writing the discharged chemical reactions (i) Dry cell (ii) Lead storage battery and (iii) Fuel cell.

*END*

# SAINIK SCHOOL IMPHAL
## SESSION 2025-26
## ENGLISH CORE (301)
## CLASS: XII
## (SUMMER VACATION HOMEWORK)

**1. What are the important tips / suggestions to follow in order to comprehend a given passage and to answer the related questions in a clear and effective manner? (200 words)**

**2. You are Kabir of class XII. Write an article analysing the significance of the French language and culture in the story "The Last Lesson" and how it relates to the theme of identity. (200 words)**

**3. Address your school assembly, presenting your views on why child labour should be eliminated and how? Refer to the hazards of children working in the bangle industry of Firozabad in the lesson "Lost Spring". (200 words)**

**4. Louisa decided to write a diary entry describing charley's visit to the Third level. Draft the diary entry in about 200 words expressing Louisa's concern and worries about her husband. (The Third Level)**

**You may begin like this**

**Dear diary,**

**I am really disturbed and worried today. On reaching home, Charley shared his experience of visiting the Third Level. I didn't believe in his words since the third level doesn't exist. I can't understand Charley's strange behaviour.**

**……………………………………………………………………………………………………………………
……………………………………………………………………………………………………………………
……………………………………………………………………………**

**5. An environmentalist came to know about the hunting spree of the Maharaja. He is deeply pained to know that Maharaja has killed many tigers.
As an ardent nature lover, he decides to write a letter to the Maharaja and urges him to live in harmony with nature. (The Tiger King)
You may begin like this**

**Dear Jung Jung Bahadur,**

**Hope you are fine. I have heard that you are on a tiger hunting spree. Your majesty, I am quite disappointed to know that the Maharaja of Pratibandapuram kingdom is very much ruthless towards wild life.**

**……………………………………………………………………………………………………………………
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**IMPORTANT NOTES:**

(i) The assignment should be done on A4 size sheets and compiled in a hard bound file/folder. And design an attractive cover for your file/folder indicating Name, Adm. No., Class, Session, Section, and Subject clearly.
(ii) All the answers should be neatly presented in your own handwriting.
(iii) Remember a well-presented 'Holiday Homework' fetches you appreciations of the teachers and the classmates.

***ENJOY THE VACATION WITH YOUR PARENTS; STAY SAFE, STAY HAPPY.***

**MR. M A HAQUE**
**MASTER (ENGLISH)**

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